Knowing our Numbers

Chapter 1

0650CH01

1.1 Introduction

Counting things is easy for us now. We can count objects in large numbers, for example, the number of students in the school, and represent them through numerals. We can also communicate large numbers using suitable number names.

It is not as if we always knew how to convey large quantities in conversation or through symbols. Many thousands years ago, people knew only small numbers. Gradually, they learnt how to handle larger numbers. They also learnt how to express large numbers in symbols. All this came through collective efforts of human beings. Their path was not easy, they struggled all along the way. In fact, the development of whole of Mathematics can be understood this way. As human beings progressed, there was greater need for development of Mathematics and as a result Mathematics grew further and faster.

We use numbers and know many things about them. Numbers help us count concrete objects. They help us to say which collection of objects is bigger and arrange them in order e.g., first, second, etc. Numbers are used in many different contexts and in many ways. Think about various situations where we use numbers. List five distinct situations in which numbers are used.

We enjoyed working with numbers in our previous classes. We have added, subtracted, multiplied and divided them. We also looked for patterns in number sequences and done many other interesting things with numbers. In this chapter, we shall move forward on such interesting things with a bit of review and revision as well.

1.2 Comparing Numbers

As we have done quite a lot of this earlier, let us see if we remember which is the greatest among these :

(i) 92, 392, 4456, 89742 — I am the greatest!

(ii) 1902, 1920, 9201, 9021, 9210 — I am the greatest!

So, we know the answers.
Discuss with your friends, how you find the number that is the greatest.

Try These

Can you instantly find the greatest and the smallest numbers in each row?

1. 382, 4972, 18, 59785, 750. Ans. 59785 is the greatest and 18 is the smallest.
2. 1473, 89423, 100, 5000, 310. Ans. ____________
3. 1834, 75284, 111, 2333, 450 . Ans. ____________
4. 2853, 7691, 9999, 12002, 124. Ans. ____________

Was that easy? Why was it easy?

We just looked at the number of digits and found the answer. The greatest number has the most thousands and the smallest is only in hundreds or in tens.

Make five more problems of this kind and give to your friends to solve.

Now, how do we compare 4875 and 3542?

This is also not very difficult.These two numbers have the same number of digits. They are both in thousands. But the digit at the thousands place in 4875 is greater than that in 3542. Therefore, 4875 is greater than 3542.

Try These

Find the greatest and the smallest numbers.

(a) 4536, 4892, 4370, 4452.
(b) 15623, 15073, 15189, 15800.
(c) 25286, 25245, 25270, 25210.
(d) 6895, 23787, 24569, 24659.

Next tell which is greater, 4875 or 4542? Here too the numbers have the same number of digits. Further, the digits at the thousands place are same in both. What do we do then? We move to the next digit, that is to the digit at the hundreds place. The digit at the hundreds place is greater in 4875 than in 4542. Therefore, 4875 is greater than 4542.

If the digits at hundreds place are also same in the two numbers, then what do we do?

Compare 4875 and 4889 ; Also compare 4875 and 4879.

1.2.1 How many numbers can you make?

Suppose, we have four digits 7, 8, 3, 5. Using these digits we want to make different 4-digit numbers in such a way that no digit is repeated in them. Thus, 7835 is allowed, but 7735 is not. Make as many 4-digit numbers as you can.

Which is the greatest number you can get? Which is the smallest number?

The greatest number is 8753 and the smallest is 3578.

Think about the arrangement of the digits in both. Can you say how the largest number is formed? Write down your procedure.

Try These

1. Use the given digits without repetition and make the greatest and smallest 4-digit numbers.

   (a) 2, 8, 7, 4 (b) 9, 7, 4, 1 (c) 4, 7, 5, 0
   (d) 1, 7, 6, 2 (e) 5, 4, 0, 3

   (**Hint :** 0754 is a 3-digit number.)

2. Now make the greatest and the smallest 4-digit numbers by using any one digit twice.

   (a) 3, 8, 7 (b) 9, 0, 5 (c) 0, 4, 9 (d) 8, 5, 1

   (**Hint :** Think in each case which digit will you use twice.)

3. Make the greatest and the smallest 4-digit numbers using any four different digits with conditions as given.

(a)	Digit 7 is always at ones place	Greatest	9	8	6	7
		Smallest	1	0	2	7

(Note, the number cannot begin with the digit 0. Why?)

(b)	Digit 4 is always at tens place	Greatest			4	
		Smallest			4	
(c)	Digit 9 is always at hundreds place	Greatest		9		
		Smallest		9		
(d)	Digit 1 is always at thousands place	Greatest	1			
		Smallest	1			

4. Take two digits, say 2 and 3. Make 4-digit numbers using both the digits equal number of times.

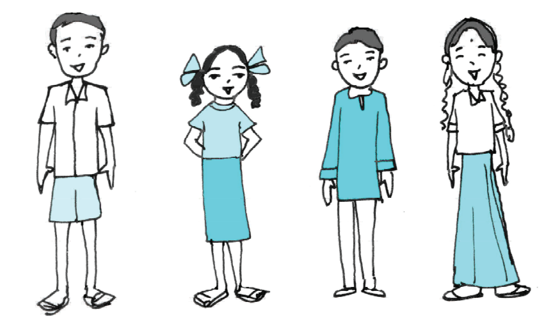
   Which is the greatest number?

   Which is the smallest number?

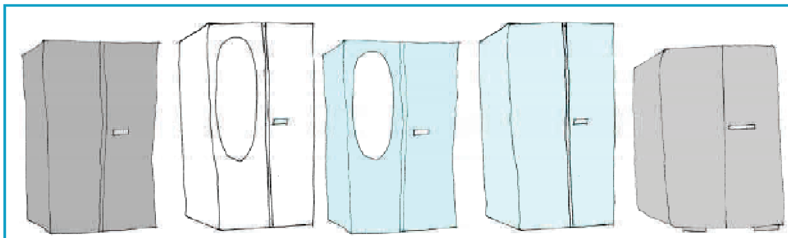
   How many different numbers can you make in all?

Stand in proper order

1. Who is the tallest?
2. Who is the shortest?
   (a) Can you arrange them in the increasing order of their heights?
   (b) Can you arrange them in the decreasing order of their heights?



Ramhari (160 cm) Dolly (154 cm) Mohan (158 cm) Shashi (159 cm)



₹ 2635 ₹ 1897 ₹ 2854 ₹ 1788 ₹ 3975

Which to buy?

Sohan and Rita went to buy an almirah. There were many almirahs available with their price tags.

(a) Can you arrange their prices in increasing order?

(b) Can you arrange their prices in decreasing order?

Try These

Think of five more situations where you compare three or more quantities.

Ascending order Ascending order means arrangement from the smallest to the greatest.

Descending order Descending order means arrangement from the greatest to the smallest.

Try These

1. Arrange the following numbers in ascending order :
   (a) 847, 9754, 8320, 571 (b) 9801, 25751, 36501, 38802
2. Arrange the following numbers in descending order :
   (a) 5000, 7500, 85400, 7861 (b) 1971, 45321, 88715, 92547

   Make ten such examples of ascending/descending order and solve them.

1.2.2 Shifting digits

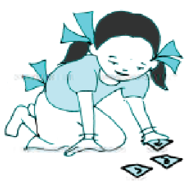
Have you thought what fun it would be if the digits in a number could shift (move) from one place to the other?

Think about what would happen to 182. It could become as large as 821 and as small as 128. Try this with 391 as well.

Now think about this. Take any 3-digit number and exchange the digit at the hundreds place with the digit at the ones place.

(a) Is the new number greater than the former one?
(b) Is the new number smaller than the former number?

Write the numbers formed in both ascending and descending order.



Before 7 9 5

Exchanging the 1st and the 3rd tiles.

After 5 9 7

If you exchange the 1st and the 3rd tiles (i.e. digits), in which case does the number become greater? In which case does it become smaller?

Try this with a 4-digit number.

1.2.3 Introducing 10,000

We know that beyond 99 there is no 2-digit number. 99 is the greatest 2-digit number. Similarly, the greatest 3-digit number is 999 and the greatest 4-digit number is 9999. What shall we get if we add 1 to 9999?

Look at the pattern :
$9 + 1 = 10 = 10 \times 1$
$99 + 1 = 100 = 10 \times 10$
$999 + 1 = 1000 = 10 \times 100$

We observe that

Greatest single digit number + 1 = smallest 2-digit number

Greatest 2-digit number + 1 = smallest 3-digit number

Greatest 3-digit number + 1 = smallest 4-digit number

We should then expect that on adding 1 to the greatest 4-digit number, we would get the smallest 5-digit number, that is 9999 + 1 = 10000.

The new number which comes next to 9999 is 10000. It is called ten thousand. Further, 10000 = 10 × 1000.

1.2.4 Revisiting place value

You have done this quite earlier, and you will certainly remember the expansion of a 2-digit number like 78 as

$78 = 70 + 8 = 7 \times 10 + 8$

Similarly, you will remember the expansion of a 3-digit number like 278 as

$278 = 200 + 70 + 8 = 2 \times 100 + 7 \times 10 + 8$

We say, here, 8 is at ones place, 7 is at tens place and 2 at hundreds place.

Later on we extended this idea to 4-digit numbers.

For example, the expansion of 5278 is

$$5278 = 5000 + 200 + 70 + 8$$
$$= 5 \times 1000 + 2 \times 100 + 7 \times 10 + 8$$

Here, 8 is at ones place, 7 is at tens place, 2 is at hundreds place and 5 is at thousands place.

With the number 10000 known to us, we may extend the idea further. We may write 5-digit numbers like

$45278 = 4 \times 10000 + 5 \times 1000 + 2 \times 100 + 7 \times 10 + 8$

We say that here 8 is at ones place, 7 at tens place, 2 at hundreds place, 5 at thousands place and 4 at ten thousands place. The number is read as forty five thousand, two hundred seventy eight. Can you now write the smallest and the greatest 5-digit numbers?

Try These

Read and expand the numbers wherever there are blanks.

Number	Number Name	Expansion
20000	twenty thousand	2×10000
26000	twenty six thousand	$2 \times 10000 + 6 \times 1000$
38400	thirty eight thousand four hundred	$3 \times 10000 + 8 \times 1000 + 4 \times 100$
65740	sixty five thousand seven hundred forty	$6 \times 10000 + 5 \times 1000 + 7 \times 100 + 4 \times 10$

89324	eighty nine thousand three hundred twenty four	8 × 10000 + 9 × 1000 + 3 × 100 + 2 × 10 + 4 × 1
50000	_______________	_______________
41000	_______________	_______________
47300	_______________	_______________
57630	_______________	_______________
29485	_______________	_______________
29085	_______________	_______________
20085	_______________	_______________
20005	_______________	_______________

Write five more 5-digit numbers, read them and expand them.

1.2.5 Introducing 1,00,000

Which is the greatest 5-digit number?

Adding 1 to the greatest 5-digit number, should give the smallest 6-digit number : 99,999 + 1 = 1,00,000

This number is named one lakh. One lakh comes next to 99,999.

10 × 10,000 = 1,00,000

We may now write 6-digit numbers in the expanded form as

2,46,853 = 2 × 1,00,000 + 4 × 10,000 + 6 × 1,000 + 8 × 100 + 5 × 10 +3 × 1

This number has 3 at ones place, 5 at tens place, 8 at hundreds place, 6 at thousands place, 4 at ten thousands place and 2 at lakh place. Its number name is two lakh forty six thousand eight hundred fifty three.

Try These

Read and expand the numbers wherever there are blanks.

Number	Number Name	Expansion
3,00,000	three lakh	3 × 1,00,000
3,50,000	three lakh fifty thousand	3 × 1,00,000 + 5 × 10,000
3,53,500	three lakh fifty three thousand five hundred	3 × 1,00,000 + 5 × 10,000 + 3 × 1000 + 5 × 100
4,57,928	_______________	_______________
4,07,928	_______________	_______________
4,00,829	_______________	_______________
4,00,029	_______________	_______________

1.2.6 Larger numbers

If we add one more to the greatest 6-digit number we get the smallest 7-digit number. It is called **ten lakh.**

Write down the greatest 6-digit number and the smallest 7-digit number. Write the greatest 7-digit number and the smallest 8-digit number. The smallest 8-digit number is called **one crore.**

Complete the pattern :

9 + 1	=	10
99 + 1	=	100
999 + 1	=	________
9,999 + 1	=	________
99,999 + 1	=	________
9,99,999 + 1	=	________
99,99,999 + 1	=	1,00,00,000

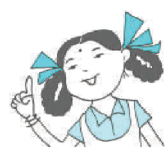
Remember

1 hundred	= 10 tens
1 thousand	= 10 hundreds
	= 100 tens
1 lakh	= 100 thousands
	= 1000 hundreds
1 crore	= 100 lakhs
	= 10,000 thousands

Try These

1. What is 10 – 1 =?
2. What is 100 – 1 =?
3. What is 10,000 – 1 =?
4. What is 1,00,000 – 1 =?
5. What is 1,00,00,000 – 1 =?

(**Hint :** Use the said pattern.)



We come across large numbers in many different situations. For example, while the number of children in your class would be a 2-digit number, the number of children in your school would be a 3 or 4-digit number.

The number of people in the nearby town would be much larger.

Is it a 5 or 6 or 7-digit number?

Do you know the number of people in your state?

How many digits would that number have?

What would be the number of grains in a sack full of wheat? A 5-digit number, a 6-digit number or more?

Try These

1. Give five examples where the number of things counted would be more than 6-digit number.
2. Starting from the greatest 6-digit number, write the previous five numbers in descending order.
3. Starting from the smallest 8-digit number, write the next five numbers in ascending order and read them.

1.2.7 An aid in reading and writing large numbers

Try reading the following numbers :

(a) 279453 (b) 5035472

(c) 152700375 (d) 40350894

Was it difficult?

Did you find it difficult to keep track?

Sometimes it helps to use indicators to read and write large numbers.

Shagufta uses indicators which help her to read and write large numbers. Her indicators are also useful in writing the expansion of numbers. For example, she identifies the digits in ones place, tens place and hundreds place in 257 by writing them under the tables O, T and H as

H	T	O	Expansion
2	5	7	$2 \times 100 + 5 \times 10 + 7 \times 1$

Similarly, for 2902,

Th	H	T	O	Expansion
2	9	0	2	$2 \times 1000 + 9 \times 100 + 0 \times 10 + 2 \times 1$

One can extend this idea to numbers upto lakh as seen in the following table. (Let us call them placement boxes). Fill the entries in the blanks left.

Number	TLakh	Lakh	TTh	Th	H	T	O	Number Name	Expansion
7,34,543	—	7	3	4	5	4	3	Seven lakh thirty four thousand five hundred forty three	----------------
32,75,829	3	2	7	5	8	2	9	--------------------	$3 \times 10,00,000 + 2 \times 1,00,000 + 7 \times 10,000 + 5 \times 1000 + 8 \times 100 + 2 \times 10 + 9$

Similarly, we may include numbers upto crore as shown below :

Number	TCr	Cr	TLakh	Lakh	TTh	Th	H	T	O	Number Name
2,57,34,543	—	2	5	7	3	4	5	4	3	....
65,32,75,829	6	5	3	2	7	5	8	2	9	Sixty five crore thirty two lakh seventy five thousand eight hundred twenty nine

You can make other formats of tables for writing the numbers in expanded form.

Use of commas

> While writing number names, we do not use commas.

You must have noticed that in writing large numbers in the sections above, we have often used commas. Commas help us in reading and writing large numbers. In our **Indian System of Numeration** we use ones, tens, hundreds, thousands and then lakhs and crores. Commas are used to mark thousands, lakhs and crores. The first comma comes after hundreds place (three digits from the right) and marks thousands. The second comma comes two digits later (five digits from the right). It comes after ten thousands place and marks lakh. The third comma comes after another two digits (seven digits from the right). It comes after ten lakh place and marks crore.

For example, 5, 08, 01, 592

3, 32, 40, 781

7, 27, 05, 062

Try reading the numbers given above. Write five more numbers in this form and read them.

International System of Numeration

In the International System of Numeration, as it is being used we have ones, tens, hundreds, thousands and then millions. One million is a thousand thousands. Commas are used to mark thousands and millions. It comes after every three digits from the right. The first comma marks thousands and the next comma marks millions. For example, the number 50,801,592 is read in the International System as fifty million eight hundred one thousand five hundred ninety two. In the Indian System, it is five crore eight lakh one thousand five hundred ninety two.

How many lakhs make a million?

How many millions make a crore?

Take three large numbers. Express them in both Indian and International Numeration systems.

Interesting fact :

To express numbers larger than a million, a billion is used in the International System of Numeration: 1 billion = 1000 million.

Do you know?
India's population increased by about
27 million during 1921-1931;
37 million during 1931-1941;
44 million during 1941-1951;
78 million during 1951-1961!

How much was the increase in population during 1991-2001? Try to find out.

Do you know what is India's population today? Try to find this too.

Try These

1. Read these numbers. Write them using placement boxes and then write their expanded forms.

   (i) 475320 (ii) 9847215 (iii) 97645310 (iv) 30458094

   (a) Which is the smallest number?

   (b) Which is the greatest number?

   (c) Arrange these numbers in ascending and descending orders.

2. Read these numbers.

   (i) 527864 (ii) 95432 (iii) 18950049 (iv) 70002509

   (a) Write these numbers using placement boxes and then using commas in Indian as well as International System of Numeration..

   (b) Arrange these in ascending and descending order.

3. Take three more groups of large numbers and do the exercise given above.

Can you help me write the numeral?

To write the numeral for a number you can follow the boxes again.

(a) Forty two lakh seventy thousand eight.

(b) Two crore ninety lakh fifty five thousand eight hundred.

(c) Seven crore sixty thousand fifty five.

Try These

1. You have the following digits 4, 5, 6, 0, 7 and 8. Using them, make five numbers each with 6 digits.

   (a) Put commas for easy reading.

   (b) Arrange them in ascending and descending order.

2. Take the digits 4, 5, 6, 7, 8 and 9. Make any three numbers each with 8 digits. Put commas for easy reading.

3. From the digits 3, 0 and 4, make five numbers each with 6 digits. Use commas.

EXERCISE 1.1

1. Fill in the blanks:
   (a) 1 lakh = _______ ten thousand.
   (b) 1 million = _______ hundred thousand.
   (c) 1 crore = _______ ten lakh.
   (d) 1 crore = _______ million.
   (e) 1 million = _______ lakh.
2. Place commas correctly and write the numerals:
   (a) Seventy three lakh seventy five thousand three hundred seven.
   (b) Nine crore five lakh forty one.
   (c) Seven crore fifty two lakh twenty one thousand three hundred two.
   (d) Fifty eight million four hundred twenty three thousand two hundred two.
   (e) Twenty three lakh thirty thousand ten.
3. Insert commas suitably and write the names according to Indian System of Numeration :
   (a) 87595762 (b) 8546283 (c) 99900046 (d) 98432701
4. Insert commas suitably and write the names according to International System of Numeration :
   (a) 78921092 (b) 7452283 (c) 99985102 (d) 48049831

1.3 Large Numbers in Practice

In earlier classes, we have learnt that we use centimetre (cm) as a unit of length. For measuring the length of a pencil, the width of a book or notebooks etc., we use centimetres. Our ruler has marks on each centimetre.

For measuring the thickness of a pencil, however, we find centimetre too big.

We use millimetre (mm) to show the thickness of a pencil.

Try These

1. How many centimetres make a kilometre?
2. Name five large cities in India. Find their population. Also, find the distance in kilometres between each pair of these cities.

(a) 10 millimetres = 1 centimetre

To measure the length of the classroom or the school building, we shall find centimetre too small. We use metre for the purpose.

(b) 1 metre = 100 centimetres

= 1000 millimetres

Even metre is too small, when we have to state distances between cities, say, Delhi and Mumbai, or Chennai and Kolkata. For this we need kilometres (km).

(c) 1 kilometre = 1000 metres

How many millimetres make 1 kilometre?

Since 1 m = 1000 mm

1 km = 1000 m = 1000 × 1000 mm = 10,00,000 mm

We go to the market to buy rice or wheat; we buy it in kilograms (kg). But items like ginger or chillies which we do not need in large quantities, we buy in grams (g).

We know 1 kilogram = 1000 grams.

Have you noticed the weight of the medicine tablets given to the sick? It is very small. It is in milligrams (mg).

1 gram = 1000 milligrams.

What is the capacity of a bucket for holding water? It is usually 20 litres (ℓ). Capacity is given in litres. But sometimes we need a smaller unit, the millilitres. A bottle of hair oil, a cleaning liquid or a soft drink have labels which give the quantity of liquid inside in millilitres (ml).

1 litre = 1000 millilitres.

Note that in all these units we have some words common like kilo, milli and centi. You should remember that among these **kilo** is the greatest and **milli** is the smallest; kilo shows 1000 times greater, milli shows 1000 times smaller, i.e. 1 kilogram = 1000 grams, 1 gram = 1000 milligrams.

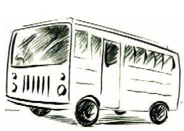
Similarly, centi shows 100 times smaller, i.e. 1 metre = 100 centimetres.

Try These

1. How many milligrams make one kilogram?
2. A box contains 2,00,000 medicine tablets each weighing 20 mg. What is the total weight of all the tablets in the box in grams and in kilograms?

Try These

1. A bus started its journey and reached different places with a speed of 60 km/hour. The journey is shown on page 14.



   (i) Find the total distance covered by the bus from A to D.

   (ii) Find the total distance covered by the bus from D to G.

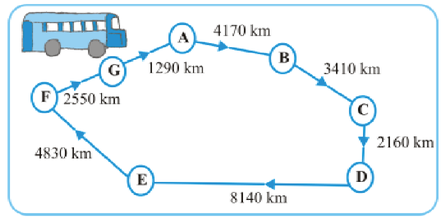
   (iii) Find the total distance covered by the bus, if it starts from A and returns back to A.

   (iv) Can you find the difference of distances from C to D and D to E?

(v) Find out the time taken by the bus to reach

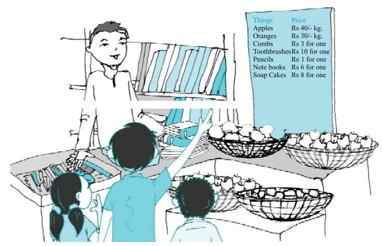
(a) A to B (b) C to D

(c) E to G (d) Total journey



2. **Raman's shop**

Things	Price
Apples	₹ 40 per kg
Oranges	₹ 30 per kg
Combs	₹ 3 for one
Tooth brushes	₹ 10 for one
Pencils	₹ 1 for one
Note books	₹ 6 for one
Soap cakes	₹ 8 for one



The sales during the last year

Apples	2457 kg
Oranges	3004 kg
Combs	22760
Tooth brushes	25367
Pencils	38530
Note books	40002
Soap cakes	20005

(a) Can you find the total weight of apples and oranges Raman sold last year?

Weight of apples = __________ kg

Weight of oranges = _________ kg

Therefore, total weight = _____ kg + _____ kg = _____ kg

Answer – The total weight of oranges and apples = _________ kg.

(b) Can you find the total money Raman got by selling apples?

(c) Can you find the total money Raman got by selling apples and oranges together?

(d) Make a table showing how much money Raman received from selling each item. Arrange the entries of amount of money received in descending order. Find the item which brought him the highest amount. How much is this amount?

We have done a lot of problems that have addition, subtraction, multiplication and division. We will try solving some more here. Before starting, look at these examples and follow the methods used.

Example 1 : Population of Sundarnagar was 2,35,471 in the year 1991. In the year 2001 it was found to be increased by 72,958. What was the population of the city in 2001?

Solution : Population of the city in 2001
= Population of the city in 1991 + Increase in population
= 2,35,471 + 72,958

Now,
```
   235471
 +  72958
 --------
   308429
 --------
```

Salma added them by writing 235471 as 200000 + 35000 + 471 and 72958 as 72000 + 958. She got the addition as 200000 + 107000 + 1429 = 308429. Mary added it as 200000 + 35000 + 400 + 71 + 72000 + 900 + 58 = 308429

Answer : Population of the city in 2001 was 3,08,429.

All three methods are correct.

Example 2 : In one state, the number of bicycles sold in the year 2002-2003 was 7,43,000. In the year 2003-2004, the number of bicycles sold was 8,00,100. In which year were more bicycles sold? and how many more?

Solution : Clearly, 8,00,100 is more than 7,43,000. So, in that state, more bicycles were sold in the year 2003-2004 than in 2002-2003.

Now,
```
   800100
 - 743000
 --------
   057100
 --------
```

Check the answer by adding
```
   743000
 +  57100
 --------
   800100   (the answer is right)
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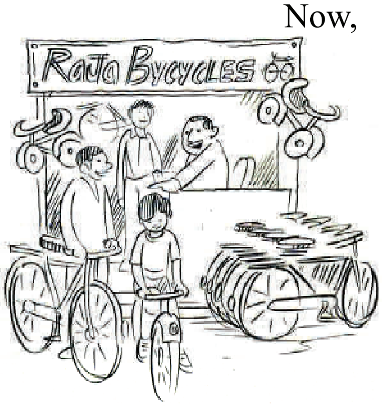


Can you think of alternative ways of solving this problem?
Answer : 57,100 more bicycles were sold in the year 2003-2004.

Example 3 : The town newspaper is published every day. One copy has 12 pages. Everyday 11,980 copies are printed. How many total pages are printed everyday?

Solution : Each copy has 12 pages. Hence, 11,980 copies will have 12 × 11,980 pages. What would this number be? More than 1,00,000 or lesser. Try to estimate.

Now,

$$\begin{array}{r} 11980 \\ \times\ 12 \\ \hline 23960 \\ +\ 119800 \\ \hline 143760 \\ \hline \end{array}$$

Answer:Everyday 1,43,760 pages are printed.

Example 4 : The number of sheets of paper available for making notebooks is 75,000. Each sheet makes 8 pages of a notebook. Each notebook contains 200 pages. How many notebooks can be made from the paper available?

Solution : Each sheet makes 8 pages.

Hence, 75,000 sheets make 8 × 75,000 pages,

Now,

$$\begin{array}{r} 75000 \\ \times\ 8 \\ \hline 600000 \\ \hline \end{array}$$

Thus, 6,00,000 pages are available for making notebooks.
Now, 200 pages make 1 notebook.
Hence, 6,00,000 pages make 6,00,000 ÷ 200 notebooks.

Now,

$$\begin{array}{r|l} & 3000 \\ \cline{2-2} 200 & 600000 \\ & -\ 600 \\ \cline{2-2} & 0000 \end{array}$$

The answer is 3,000 notebooks.

EXERCISE 1.2

1. A book exhibition was held for four days in a school. The number of tickets sold at the counter on the first, second, third and final day was respectively 1094, 1812, 2050 and 2751. Find the total number of tickets sold on all the four days.
2. Shekhar is a famous cricket player. He has so far scored 6980 runs in test matches. He wishes to complete 10,000 runs. How many more runs does he need?
3. In an election, the successful candidate registered 5,77,500 votes and his nearest rival secured 3,48,700 votes. By what margin did the successful candidate win the election?
4. Kirti bookstore sold books worth ₹ 2,85,891 in the first week of June and books worth ₹ 4,00,768 in the second week of the month. How much was the sale for the two weeks together? In which week was the sale greater and by how much?

5. Find the difference between the greatest and the least 5-digit number that can be written using the digits 6, 2, 7, 4, 3 each only once.
6. A machine, on an average, manufactures 2,825 screws a day. How many screws did it produce in the month of January 2006?
7. A merchant had ₹ 78,592 with her. She placed an order for purchasing 40 radio sets at ₹ 1200 each. How much money will remain with her after the purchase?
8. A student multiplied 7236 by 65 instead of multiplying by 56. By how much was his answer greater than the correct answer? (**Hint:** Do you need to do both the multiplications?)
9. To stitch a shirt, 2 m 15 cm cloth is needed. Out of 40 m cloth, how many shirts can be stitched and how much cloth will remain?

   (Hint: convert data in cm.)
10. Medicine is packed in boxes, each weighing 4 kg 500g. How many such boxes can be loaded in a van which cannot carry beyond 800 kg?
11. The distance between the school and a student's house is 1 km 875 m. Everyday she walks both ways. Find the total distance covered by her in six days.
12. A vessel has 4 litres and 500 ml of curd. In how many glasses, each of 25 ml capacity, can it be filled?

1.3.1 Estimation

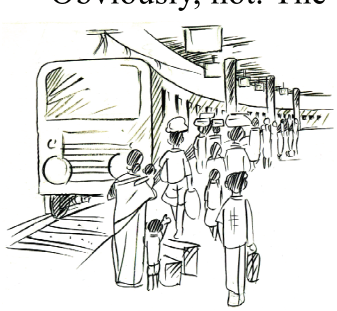
News

1. India drew with Pakistan in a hockey match watched by approximately 51,000 spectators in the stadium and 40 million television viewers world wide.
2. Approximately, 2000 people were killed and more than 50000 injured in a cyclonic storm in coastal areas of India and Bangladesh.
3. Over 13 million passengers are carried over 63,000 kilometre route of railway track every day.

Can we say that there were exactly as many people as the numbers quoted in these news items? For example,

In (1), were there exactly 51,000 spectators in the stadium? or did exactly 40 million viewers watched the match on television?

Obviously, not. The word *approximately* itself shows that the number of people were near about these numbers. Clearly, 51,000 could be 50,800 or 51,300 but not 70,000. Similarly, 40 million implies much more than 39 million but quite less than 41 million but certainly not 50 million.



The quantities given in the examples above are not exact counts, but are estimates to give an idea of the quantity.

Discuss what each of these can suggest.

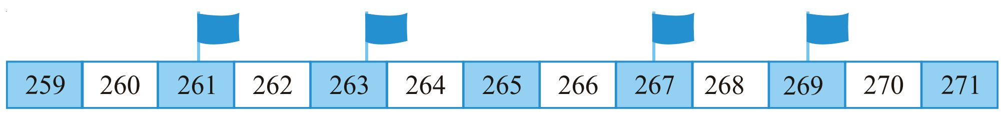
Where do we approximate? Imagine a big celebration at your home. The first thing you do is to find out roughly how many guests may visit you. Can you get an idea of the exact number of visitors? It is practically impossible.

The finance minister of the country presents a budget annually. The minister provides for certain amount under the head 'Education'. Can the amount be absolutely accurate? It can only be a reasonably good estimate of the expenditure the country needs for education during the year.

Think about the situations where we need to have the exact numbers and compare them with situations where you can do with only an approximately estimated number. Give three examples of each of such situations.

1.3.2 Estimating to the nearest tens by rounding off

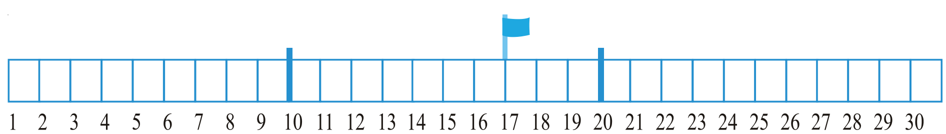
Look at the following :



(a) Find which flags are closer to 260.

(b) Find the flags which are closer to 270.

Locate the numbers 10,17 and 20 on your ruler. Is 17 nearer to 10 or 20? The gap between 17 and 20 is smaller when compared to the gap between 17 and 10. So, we round off 17 as 20, correct to the nearest tens.



Now consider 12, which also lies between 10 and 20. However, 12 is closer to 10 than to 20. So, we round off 12 to 10, correct to the nearest tens.

How would you round off 76 to the nearest tens? Is it not 80?

We see that the numbers 1,2,3 and 4 are nearer to 0 than to 10. So, we round off 1, 2, 3 and 4 as 0. Number 6, 7, 8, 9 are nearer to 10, so, we round them off as 10. Number 5 is equidistant from both 0 and 10; it is a common practice to round it off as 10.

Try These

Round these numbers to the nearest tens.

28	32	52	41	39	48
64	59	99	215	1453	2936

1.3.3 Estimating to the nearest hundreds by rounding off

Is 410 nearer to 400 or to 500?

410 is closer to 400, so it is rounded off to 400, correct to the nearest hundred.

889 lies between 800 and 900.

It is nearer to 900, so it is rounded off as 900 correct to nearest hundred.

Numbers 1 to 49 are closer to 0 than to 100, and so are rounded off to 0.

Numbers 51 to 99 are closer to 100 than to 0, and so are rounded off to 100. Number 50 is equidistant from 0 and 100 both. It is a common practice to round it off as 100.

Check if the following rounding off is correct or not :

841 $\longrightarrow$ 800; 9537 $\longrightarrow$ 9500; 49730 $\longrightarrow$ 49700;

2546 $\longrightarrow$ 2500; 286 $\longrightarrow$ 200; 5750 $\longrightarrow$ 5800;

168 $\longrightarrow$ 200; 149 $\longrightarrow$ 100; 9870 $\longrightarrow$ 9800.

Correct those which are wrong.

1.3.4 Estimating to the nearest thousands by rounding off

We know that numbers 1 to 499 are nearer to 0 than to 1000, so these numbers are rounded off as 0.

The numbers 501 to 999 are nearer to 1000 than 0 so they are rounded off as 1000.

Number 500 is also rounded off as 1000.

Check if the following rounding off is correct or not :

2573 $\longrightarrow$ 3000; 53552 $\longrightarrow$ 53000;

6404 $\longrightarrow$ 6000; 65437 $\longrightarrow$ 65000;

7805 $\longrightarrow$ 7000; 3499 $\longrightarrow$ 4000.

Correct those which are wrong.

Try These

Round off the given numbers to the nearest tens, hundreds and thousands.

Given Number	Approximate to Nearest	Rounded Form
75847	Tens	________
75847	Hundreds	________
75847	Thousands	________
75847	Ten thousands	________

1.3.5 Estimating outcomes of number situations

How do we add numbers? We add numbers by following the algorithm (i.e. the given method) systematically. We write the numbers taking care that the digits in the same place (ones, tens, hundreds etc.) are in the same column. For example, 3946 + 6579 + 2050 is written as —

Th	H	T	O
3	9	4	6
6	5	7	9
+ 2	0	5	0

We add the column of ones and if necessary carry forward the appropriate number to the tens place as would be in this case. We then add the tens column and this goes on. Complete the rest of the sum yourself. This procedure takes time.

There are many situations where we need to find answers more quickly. For example, when you go to a fair or the market, you find a variety of attractive things which you want to buy. You need to quickly decide what you can buy. So, you need to estimate the amount you need. It is the sum of the prices of things you want to buy.

A trader is to receive money from two sources. The money he is to receive is ₹ 13,569 from one source and ₹ 26,785 from another. He has to pay ₹ 37,000 to someone else by the evening. He rounds off the numbers to their nearest thousands and quickly works out the rough answer. He is happy that he has enough money.

Do you think he would have enough money? Can you tell without doing the exact addition/subtraction?

Sheila and Mohan have to plan their monthly expenditure. They know their monthly expenses on transport, on school requirements, on groceries, on milk, and on clothes and also on other regular expenses. This month they have to go for visiting and buying gifts. They estimate the amount they would spend on all this and then add to see, if what they have, would be enough.

Would they round off to thousands as the trader did?

Think and discuss five more situations where we have to estimate sums or remainders.

Did we use rounding off to the same place in all these?

There are no rigid rules when you want to estimate the outcomes of numbers. The procedure depends on the degree of accuracy required and how quickly the estimate is needed. The most important thing is, how sensible the guessed answer would be.

1.3.6 To estimate sum or difference

As we have seen above we can round off a number to any place. The trader rounded off the amounts to the nearest thousands and was satisfied that he had enough. So, when you estimate any sum or difference, you should have an idea of why you need to round off and therefore the place to which you would round off. Look at the following examples.

Example 5 : Estimate: 5,290 + 17,986.

Solution : You find 17,986 > 5,290.

Round off to thousands.

17,986 is rounds off to		18,000
+5,290 is rounds off to		+ 5,000
Estimated sum	=	23,000

Does the method work? You may attempt to find the actual answer and verify if the estimate is reasonable.

Example 6 : Estimate: 5,673 – 436.

Solution : To begin with we round off to thousands. (Why?)

5,673 rounds off to		6,000
– 436 rounds off to		– 0
Estimated difference	=	6,000

This is not a reasonable estimate. Why is this not reasonable?

To get a closer estimate, let us try rounding each number to hundreds.

5,673 rounds off to	5,700
– 436 rounds off to	– 400
Estimated difference =	5,300

This is a better and more meaningful estimate.

1.3.7 To estimate products

How do we estimate a product?

What is the estimate for 19 × 78?

It is obvious that the product is less than 2000. Why?

If we approximate 19 to the nearest tens, we get 20 and then approximate 78 to nearest tens, we get 80 and 20 × 80 = 1600

Look at 63 × 182

If we approximate both to the nearest hundreds we get 100 × 200 = 20,000. This is much larger than the actual product. So, what do we do? To get a more reasonable estimate, we try rounding off 63 to the nearest 10, i.e. 60, and also 182 to the nearest ten, i.e. 180. We get 60 × 180 or 10,800. This is a good estimate, but is not quick enough.

If we now try approximating 63 to 60 and 182 to the nearest hundred, i.e. 200, we get 60 × 200, and this number 12,000 is a quick as well as good estimate of the product.

Try These

Estimate the following products :

(a) 87 × 313

(b) 9 × 795

(c) 898 × 785

(d) 958 × 387

Make five more such problems and solve them.

The general rule that we can make is, therefore, *Round off each factor to its greatest place, then multiply the rounded off factors*. Thus, in the above example, we rounded off 63 to tens and 182 to hundreds.

Now, estimate 81 × 479 using this rule :

479 is rounded off to 500 (rounding off to hundreds),

and 81 is rounded off to 80 (rounding off to tens).

The estimated product = 500 × 80 = 40,000

An important use of estimates for you will be to check your answers. Suppose, you have done the multiplication 37 × 1889, but are not sure about your answer. A quick and reasonable estimate of the product will be 40 × 2000 i.e. 80,000. If your answer is close to 80,000, it is probably right. On the other hand, if it is close to 8000 or 8,00,000, something is surely wrong in your multiplication.

Same general rule may be followed by addition and subtraction of two or more numbers.

EXERCISE 1.3

1. Estimate each of the following using general rule:

   (a) 730 + 998 (b) 796 – 314 (c) 12,904 +2,888 (d) 28,292 – 21,496

   Make ten more such examples of addition, subtraction and estimation of their outcome.
2. Give a rough estimate (by rounding off to nearest hundreds) and also a closer estimate (by rounding off to nearest tens) :

   (a) 439 + 334 + 4,317 (b) 1,08,734 – 47,599 (c) 8325 – 491

   (d) 4,89,348 – 48,365

   Make four more such examples.
3. Estimate the following products using general rule:

   (a) 578 × 161 (b) 5281 × 3491 (c) 1291 × 592 (d) 9250 × 29

   Make four more such examples.

1.4 Using Brackets

Meera bought 6 notebooks from the market and the cost was ₹ 10 per notebook. Her sister Seema also bought 7 notebooks of the same type. Find the total money they paid.

Seema calculated the amount like this		**Meera calculated the amount like this**
6 × 10 + 7 × 10		6 + 7 =13
= 60 + 70		
= 130	and	13 × 10 = 130
Ans. ₹ 130		Ans. ₹ 130

You can see that Seema's and Meera's ways to get the answer are a bit different. But both give the correct result. Why?

Seema says, what Meera has done is 7 + 6 × 10.

Appu points out that 7 + 6 × 10 = 7 + 60 = 67. Thus, this is not what Meera had done. All the three students are confused.

To avoid confusion in such cases we may use brackets. We can pack the numbers 6 and 7 together using a bracket, indicating that the pack is to be treated as a single number. Thus, the answer is found by (6 + 7) × 10 = 13 × 10.

This is what Meera did. She first added 6 and 7 and then multiplied the sum by 10.

This clearly tells us : *First, turn everything inside the brackets ( ) into a single number and then do the operation outside which in this case is to*

Try These

1. Write the expressions for each of the following using brackets.
   (a) Four multiplied by the sum of nine and two.
   (b) Divide the difference of eighteen and six by four.
   (c) Forty five divided by three times the sum of three and two.
2. Write three different situations for $(5 + 8) \times 6$.
   (One such situation is : Sohani and Reeta work for 6 days; Sohani works 5 hours a day and Reeta 8 hours a day. How many hours do both of them work in a week?)
3. Write five situations for the following where brackets would be necessary. (a) $7(8 – 3)$ (b) $(7 + 2) (10 – 3)$

multiply by 10.

1.4.1 Expanding brackets

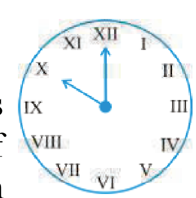
Now, observe how use of brackets allows us to follow our procedure systematically. Do you think that it will be easy to keep a track of what steps we have to follow without using brackets?

(i) $7 \times 109 = 7 \times (100 + 9) = 7 \times 100 + 7 \times 9 = 700 + 63 = 763$

(ii) $102 \times 103 = (100 + 2) \times (100 + 3) = (100 + 2) \times 100 + (100 + 2) \times 3$
$= 100 \times 100 + 2 \times 100 + 100 \times 3 + 2 \times 3$
$= 10{,}000 + 200 + 300 + 6 = 10{,}000 + 500 + 6$
$= 10{,}506$

(iii) $17 \times 109 = (10 + 7) \times 109 = 10 \times 109 + 7 \times 109$
$= 10 \times (100 + 9) + 7 \times (100 + 9)$
$= 10 \times 100 + 10 \times 9 + 7 \times 100 + 7 \times 9$
$= 1000 + 90 + 700 + 63 = 1{,}790 + 63$
$= 1{,}853$

1.5 Roman Numerals



We have been using the Hindu-Arabic numeral system so far. This is not the only system available. One of the early systems of writing numerals is the system of Roman numerals. This system is still used in many places.

For example, we can see the use of Roman numerals in clocks; it is also used for classes in the school time table etc.

Find three other examples, where Roman numerals are used.

The Roman numerals :

I, II, III, IV, V, VI, VII, VIII, IX, X

denote 1,2,3,4,5,6,7,8,9 and 10 respectively. This is followed by XI for 11, XII for 12,... till XX for 20. Some more Roman numerals are :

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

The rules for the system are :

(a) If a symbol is repeated, its value is added as many times as it occurs:

i.e. II is equal 2, XX is 20 and XXX is 30.

(b) A symbol is not repeated more than three times. But the symbols V, L and D are never repeated.

(c) If a symbol of smaller value is written to the right of a symbol of greater value, its value gets added to the value of greater symbol.

VI = 5 + 1 = 6, XII = 10 + 2 = 12

and LXV = 50 + 10 + 5 = 65

(d) If a symbol of smaller value is written to the left of a symbol of greater value, its value is subtracted from the value of the greater symbol.

IV = 5 – 1 = 4, IX = 10 – 1 = 9

XL = 50 – 10 = 40, XC = 100 – 10 = 90

(e) The symbols V, L and D are never written to the left of a symbol of greater value, i.e. V, L and D are never subtracted.

The symbol I can be subtracted from V and X only.

The symbol X can be subtracted from L, M and C only.

Following these rules we get,

1 = I	10 = X	100 = C
2 = II	20 = XX	
3 = III	30 = XXX	
4 = IV	40 = XL	
5 = V	50 = L	
6 = VI	60 = LX	
7 = VII	70 = LXX	
8 = VIII	80 = LXXX	
9 = IX	90 = XC	

Try These

Write in Roman numerals.

1. 73
2. 92

(a) Write in Roman numerals the missing numbers in the table.

(b) XXXX, VX, IC, XVV are not written. Can you tell why?

Example 7 : Write in Roman Numerals (a) 69 (b) 98.

Solution : (a) 69 = 60 + 9
= (50 + 10) + 9
= LX + IX
= LX IX

(b) 98 = 90 + 8
= (100 – 10) + 8
= XC + VIII
= XCVIII

What have we discussed?

1. Given two numbers, one with more digits is the greater number. If the number of digits in two given numbers is the same, that number is larger, which has a greater leftmost digit. If this digit also happens to be the same, we look at the next digit and so on.
2. In forming numbers from given digits, we should be careful to see if the conditions under which the numbers are to be formed are satisfied. Thus, to form the greatest four digit number from 7, 8, 3, 5 without repeating a single digit, we need to use all four digits, the greatest number can have only 8 as the leftmost digit.
3. The smallest four digit number is 1000 (one thousand). It follows the largest three digit number 999. Similarly, the smallest five digit number is 10,000. It is ten thousand and follows the largest four digit number 9999.

   Further, the smallest six digit number is 100,000. It is one lakh and follows the largest five digit number 99,999. This carries on for higher digit numbers in a similar manner.
4. Use of commas helps in reading and writing large numbers. In the Indian system of numeration we have commas after 3 digits starting from the right and thereafter every 2 digits. The commas after 3, 5 and 7 digits separate thousand, lakh and crore respectively. In the International system of numeration commas are placed after every 3 digits starting from the right. The commas after 3 and 6 digits separate thousand and million respectively.
5. Large numbers are needed in many places in daily life. For example, for giving number of students in a school, number of people in a village or town, money paid or received in large transactions (paying and selling), in measuring large distances say betwen various cities in a country or in the world and so on.
6. Remember kilo shows 1000 times larger, Centi shows 100 times smaller and milli shows 1000 times smaller, thus, 1 kilometre = 1000 metres, 1 metre = 100 centimetres or 1000 millimetres etc.
7. There are a number of situations in which we do not need the exact quantity but need only a reasonable guess or an estimate. For example, while stating how many spectators watched a particular international hockey match, we state the approximate number,

say 51,000, we do not need to state the exact number.

8. Estimation involves approximating a quantity to an accuracy required. Thus, 4117 may be approximated to 4100 or to 4000, i.e. to the nearest hundred or to the nearest thousand depending on our need.
9. In number of situations, we have to estimate the outcome of number operations. This is done by rounding off the numbers involved and getting a quick, rough answer.
10. Estimating the outcome of number operations is useful in checking answers.
11. Use of brackets allows us to avoid confusion in the problems where we need to carry out more than one number operation.
12. We use the Hindu-Arabic system of numerals. Another system of writing numerals is the Roman system.

Whole Numbers

Chapter 2

2.1 Introduction

As we know, we use 1, 2, 3, 4,... when we begin to count. They come naturally when we start counting. Hence, mathematicians call the counting numbers as Natural numbers.

Predecessor and successor

Given any natural number, you can add 1 to that number and get the next number i.e. you get its successor.

The successor of 16 is 16 + 1 = 17, that of 19 is 19 +1 = 20 and so on.

The number 16 comes before 17, we say that the predecessor of 17 is 17–1=16, the predecessor of 20 is 20 – 1 = 19, and so on.

The number 3 has a predecessor and a successor. What about 2? The successor is 3 and the predecessor is 1. Does 1 have both a successor and a predecessor?

Try These

1. Write the predecessor and successor of 19; 1997; 12000; 49; 100000.
2. Is there any natural number that has no predecessor?
3. Is there any natural number which has no successor? Is there a last natural number?

We can count the number of children in our school; we can also count the number of people in a city; we can count the number of people in India. The number of people in the whole world can also be counted. We may not be able to count the number of stars in the sky or the number of hair on our heads but if we are able, there would be a number for them also. We can then add one more to such a number and get a larger number. In that case we can even write the number of hair on two heads taken together.

It is now perhaps obvious that there is no largest number. Apart from these questions shared above, there are many others that can come to our mind when we work with natural numbers. You can think of a few such questions and discuss them with your friends. You may not clearly know the answers to many of them !

2.2 Whole Numbers

We have seen that the number 1 has no predecessor in natural numbers. To the collection of natural numbers we add zero as the predecessor for 1.

> The natural numbers along with zero form the collection of whole numbers.

Try These

1. Are all natural numbers also whole numbers?
2. Are all whole numbers also natural numbers?
3. Which is the greatest whole number?

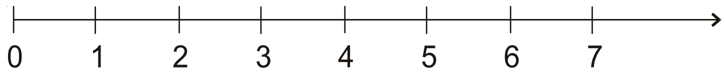
In your previous classes you have learnt to perform all the basic operations like addition, subtraction, multiplication and division on numbers. You also know how to apply them to problems. Let us try them on a number line. Before we proceed, let us find out what a number line is!

2.3 The Number Line

Draw a line. Mark a point on it. Label it 0. Mark a second point to the right of 0. Label it 1.

The distance between these points labelled as 0 and 1 is called unit distance. On this line, mark a point to the right of 1 and at unit distance from 1 and label it 2. In this way go on labelling points at unit distances as 3, 4, 5,... on the line. You can go to any whole number on the right in this manner.

This is a number line for the whole numbers.



What is the distance between the points 2 and 4? Certainly, it is 2 units. Can you tell the distance between the points 2 and 6, between 2 and 7?

On the number line you will see that the number 7 is on the right of 4. This number 7 is greater than 4, i.e. $7 > 4$. The number 8 lies on the right of 6 and 8 > 6. These observations help us to say that, out of any two whole numbers, the number on the right of the other number is the greater number. We can also say that whole number on left is the smaller number.

For example, 4 < 9; 4 is on the left of 9. Similarly, 12 > 5; 12 is to the right of 5.

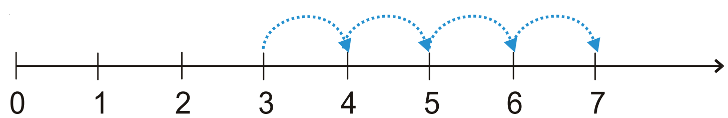
What can you say about 10 and 20?

Mark 30, 12, 18 on the number line. Which number is at the farthest left? Can you say from 1005 and 9756, which number would be on the right relative to the other number.

Place the successor of 12 and the predecessor of 7 on the number line.

Addition on the number line

Addition of whole numbers can be shown on the number line. Let us see the addition of 3 and 4.



Start from 3. Since we add 4 to this number so we make 4 jumps to the right; from 3 to 4, 4 to 5, 5 to 6 and 6 to 7 as shown above. The tip of the last arrow in the fourth jump is at 7.

The sum of 3 and 4 is 7, i.e. 3 + 4 = 7.

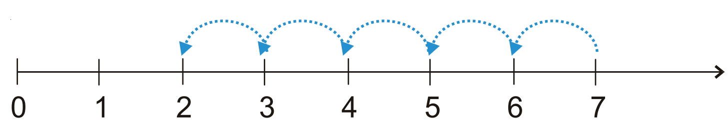
Try These

Find 4 + 5; 2 + 6; 3 + 5 and 1+6 using the number line.

Subtraction on the number line

The subtraction of two whole numbers can also be shown on the number line.

Let us find 7 – 5.



Start from 7. Since 5 is being subtracted, so move towards left with 1 jump of 1 unit. Make 5 such jumps. We reach the point 2. We get 7 – 5 = 2.

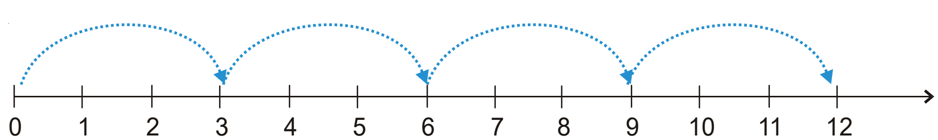
Try These

Find 8 – 3; 6 – 2; 9 – 6 using the number line.

Multiplication on the number line

We now see the multiplication of whole numbers on the number line.

Let us find 4 × 3.



Start from 0, move 3 units at a time to the right, make such 4 moves. Where do you reach? You will reach 12. So, we say, $3 \times 4 = 12$.

Try These

Find 2×6; 3×3; 4×2 using the number line.

EXERCISE 2.1

1. Write the next three natural numbers after 10999.
2. Write the three whole numbers occurring just before 10001.
3. Which is the smallest whole number?
4. How many whole numbers are there between 32 and 53?
5. Write the successor of :

   (a) 2440701 (b) 100199 (c) 1099999 (d) 2345670
6. Write the predecessor of :

   (a) 94 (b) 10000 (c) 208090 (d) 7654321
7. In each of the following pairs of numbers, state which whole number is on the left of the other number on the number line. Also write them with the appropriate sign (>, <) between them.

   (a) 530, 503 (b) 370, 307 (c) 98765, 56789 (d) 9830415, 10023001
8. Which of the following statements are true (T) and which are false (F) ?

   (a) Zero is the smallest natural number. (b) 400 is the predecessor of 399.

   (c) Zero is the smallest whole number. (d) 600 is the successor of 599.

   (e) All natural numbers are whole numbers.

   (f) All whole numbers are natural numbers.

   (g) The predecessor of a two digit number is never a single digit number.

   (h) 1 is the smallest whole number.

   (i) The natural number 1 has no predecessor.

   (j) The whole number 1 has no predecessor.

   (k) The whole number 13 lies between 11 and 12.

   (l) The whole number 0 has no predecessor.

   (m) The successor of a two digit number is always a two digit number.

2.4 Properties of Whole Numbers

When we look into various operations on numbers closely, we notice several properties of whole numbers. These properties help us to understand the numbers better. Moreover, they make calculations under certain operations very simple.

Do This

Let each one of you in the class take any two whole numbers and add them. Is the result always a whole number?

Your additions may be like this:

7	+	8	=	15, a whole number
5	+	5	=	10, a whole number
0	+	15	=	15, a whole number
.	+	.	=	…
.	+	.	=	…

Try with five other pairs of numbers. Is the sum always a whole number?

Did you find a pair of whole numbers whose sum is not a whole number? Hence, we say that sum of any two whole numbers is a whole number i.e. the collection of whole numbers is closed under addition. This property is known as the closure property for addition of whole numbers.

Are the whole numbers closed under multiplication too? How will you check it?

Your multiplications may be like this :

7	×	8	=	56, a whole number
5	×	5	=	25, a whole number
0	×	15	=	0, a whole number
.	×	.	=	…
.	×	.	=	…

The multiplication of two whole numbers is also found to be a whole number again. We say that the system of whole numbers is closed under multiplication.

Closure property : Whole numbers are closed under addition and also under multiplication.

Think, discuss and write

1. The whole numbers are not closed under subtraction. Why?
   Your subtractions may be like this :

6	–	2	=	4, a whole number
7	–	8	=	?, not a whole number
5	–	4	=	1, a whole number
3	–	9	=	?, not a whole number

Take a few examples of your own and confirm.

2. Are the whole numbers closed under division? No. Observe this table :

8	÷	4	=	2, a whole number
5	÷	7	=	$\frac{5}{7}$, not a whole number
12	÷	3	=	4, a whole number
6	÷	5	=	$\frac{6}{5}$, not a whole number

Justify it by taking a few more examples of your own.

Division by zero

Division by a number means subtracting that number repeatedly.

Let us find 8 ÷ 2.

$$\begin{array}{rl} 8 & \\ -\ 2 & \text{........ 1} \\ \hline 6 & \\ -\ 2 & \text{........ 2} \\ \hline 4 & \\ -\ 2 & \text{........ 3} \\ \hline 2 & \\ -\ 2 & \text{........ 4} \\ \hline 0 & \end{array}$$

Subtract 2 again and again from 8.

After how many moves did we reach 0? In four moves.

So, we write 8 ÷ 2 = 4.

Using this, find 24 ÷ 8; 16 ÷ 4.

Let us now try 2 ÷ 0.

$$\begin{array}{rl} 2 & \\ -\ 0 & \text{........ 1} \\ \hline 2 & \\ -\ 0 & \text{........ 2} \\ \hline 2 & \\ -\ 0 & \text{........ 3} \\ \hline 2 & \\ -\ 0 & \text{........ 4} \\ \hline 2 & \\ \vdots & \quad \vdots \end{array}$$

In every move we get 2 again !

Will this ever stop? No.

We say 2 ÷ 0 is not defined.

Let us try 7 ÷ 0

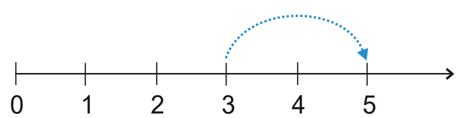
$$\begin{array}{rl} & 7 \\ - & 0 \quad \text{........ 1} \\ \hline & 7 \\ - & 0 \quad \text{........ 2} \\ \hline & 7 \\ - & 0 \quad \text{........ 3} \\ \hline & 7 \\ & \vdots \end{array}$$

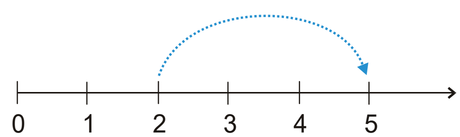
Again, we never get 0 at any stage of subtraction.
We say 7 ÷ 0 is not defined.
Check it for 5 ÷ 0, 16 ÷ 0.

Division of a whole number by 0 is not defined.

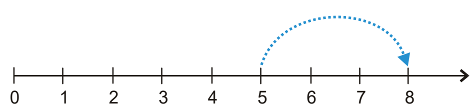
Commutativity of addition and multiplication

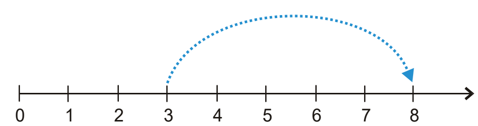
What do the following number line diagrams say?





In both the cases we reach 5. So, 3 + 2 is same as 2 + 3.
Similarly, 5 + 3 is same as 3 + 5.





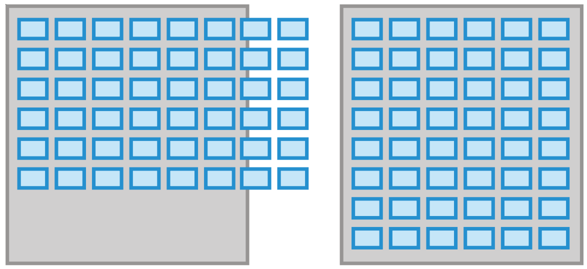
Try it for 4 + 6 and 6 + 4.

Is this true when any two whole numbers are added? Check it. You will not get any pair of whole numbers for which the sum is different when the order of addition is changed.

You can add two whole numbers in any order.

We say that addition is commutative for whole numbers. This property is known as commutativity for addition.

Discuss with your friends



You have a small party at home. You want to arrange 6 rows of chairs with 8 chairs in each row for the visitors. The number of chairs you will need is 6 × 8. You find that the room is not wide enough to accommodate rows of 8 chairs. You decide to have 8 rows of chairs with 6 chairs in each row. How many chairs do you require now? Will you require more number of chairs?

Is there a commutative property of multiplication?

Multiply numbers 4 and 5 in different orders.

You will observe that $4 \times 5 = 5 \times 4$.

Is it true for the numbers 3 and 6; 5 and 7 also?

> You can multiply two whole numbers in any order.

We say multiplication is **commutative** for whole numbers.

Thus, addition and multiplication are commutative for whole numbers.

Verify :

(i) Subtraction is not commutative for whole numbers. Use at least three different pairs of numbers to verify it.

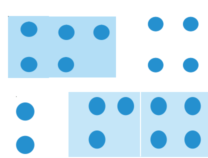
(ii) Is $(6 \div 3)$ same as $(3 \div 6)$?
Justify it by taking few more combinations of whole numbers.

Associativity of addition and multiplication

Observe the following diagrams :

(a) $(2 + 3) + 4 = 5 + 4 = 9$

(b) $2 + (3 + 4) = 2 + 7 = 9$



In (a) above, you can add 2 and 3 first and then add 4 to the sum and in (b) you can add 3 and 4 first and then add 2 to the sum.

Are not the results same?

We also have, $(5 + 7) + 3 = 12 + 3 = 15$ and $5 + (7 + 3) = 5 + 10 = 15$.

So, $(5 + 7) + 3 = 5 + (7 + 3)$

This is associativity of addition for whole numbers. Check it for the numbers 2, 8 and 6.

Notice how we grouped the numbers for convenience of adding.

Example 1 : Add the numbers 234, 197 and 103.

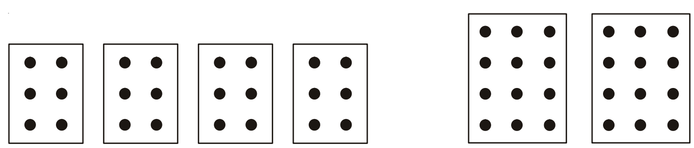
Solution : 234 + 197 + 103 = 234 + (197 + 103)
= 234 + 300 = 534

Play this game

You and your friend can play this.

You call a number from 1 to 10. Your friend now adds to this number any number from 1 to 10. Then it is your turn. You both play alternately. The winner is the one who reaches 100 first. If you always want to win the game, what will be your strategy or plan?

Observe the multiplication fact illustrated by the following diagrams (Fig 2.1).



(a) (b)

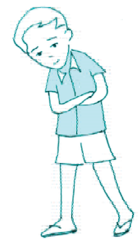
Fig 2.1

Count the number of dots in Fig 2.1 (a) and Fig 2.1 (b). What do you get? The number of dots is the same. In Fig 2.1 (a), we have 2 × 3 dots in each box. So, the total number of dots is (2 × 3) × 4 = 24.

In Fig 2.1 (b), each box has 3 × 4 dots, so in all there are 2 × (3 × 4) = 24 dots. Thus, (2 × 3) × 4 = 2 × (3 × 4). Similarly, you can see that (3 × 5) × 4 = 3 × (5 × 4)

Try this for (5 × 6) × 2 and 5 × (6 × 2); (3 × 6) × 4 and 3 × (6 × 4).

This is associative property for multiplication of whole numbers.



Think on and find :

Which is easier and why?

(a) $(6 \times 5) \times 3$ or $6 \times (5 \times 3)$

(b) $(9 \times 4) \times 25$ or $9 \times (4 \times 25)$

Example 2 : Find $14 + 17 + 6$ in two ways.

Solution : $(14 + 17) + 6 = 31 + 6 = 37$,

$14 + 17 + 6 = 14 + 6 + 17 = (14 + 6) + 17 = 20 + 17 = 37$

Here, you have used a combination of associative and commutative properties for addition.

Do you think using the commutative and the associative property has made the calculation easier?

The associative property of multiplication is very useful in the following types of sums.

Try These

Find : $7 + 18 + 13$; $16 + 12 + 4$

Example 3 : Find 12×35.

Solution : $12 \times 35 = (6 \times 2) \times 35 = 6 \times (2 \times 35) = 6 \times 70 = 420$.

In the above example, we have used associativity to get the advantage of multiplying the smallest even number by a multiple of 5.

Example 4 : Find $8 \times 1769 \times 125$

Solution : $8 \times 1769 \times 125 = 8 \times 125 \times 1769$

(What property do you use here?)

$= (8 \times 125) \times 1769$

$= 1000 \times 1769 = 17,69,000$.

Try These

Find :

$25 \times 8358 \times 4$;

$625 \times 3759 \times 8$

Think, discuss and write

Is $(16 \div 4) \div 2 = 16 \div (4 \div 2)$?

Is there an associative property for division? No.

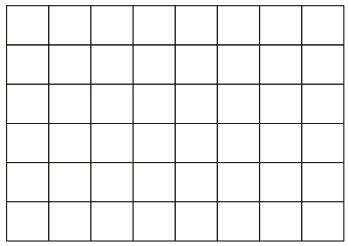
Discuss with your friends. Think of $(28 \div 14) \div 2$ and $28 \div (14 \div 2)$.

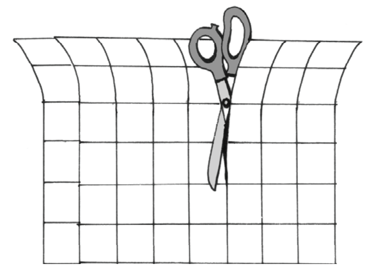
Do This

Distributivity of multiplication over addition

Take a graph paper of size 6 cm by 8 cm having squares of size 1 cm × 1 cm.

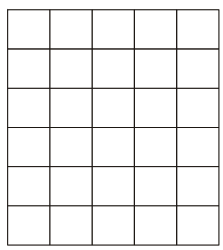
How many squares do you have in all?

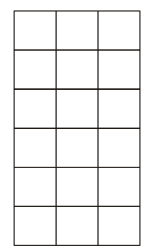




Is the number 6×8?

Now cut the sheet into two pieces of sizes 6 cm by 5 cm and 6 cm by 3 cm, as shown in the figure.





Number of squares : Is it 6×5? Number of squares : Is it 6×3?

In all, how many squares are there in both the pieces?

Is it $(6 \times 5) + (6 \times 3)$? Does it mean that $6 \times 8 = (6 \times 5) + (6 \times 3)$?

But, $6 \times 8 = 6 \times (5 + 3)$

Does this show that $6 \times (5 + 3) = (6 \times 5) + (6 \times 3)$?

Similarly, you will find that $2 \times (3 + 5) = (2 \times 3) + (2 \times 5)$

This is known as distributivity of multiplication over addition.

find using distributivity : $4 \times (5 + 8)$; $6 \times (7 + 9)$; $7 \times (11 + 9)$.

Think, discuss and write

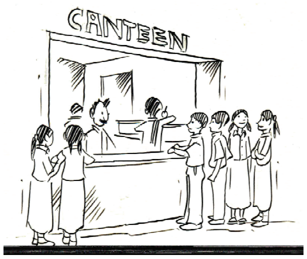
Observe the following multiplication and discuss whether we use here the idea of distributivity of multiplication over addition.

425			
×136			
2550	←	425×6	(multiplication by 6 ones)
12750	←	425×30	(multiplication by 3 tens)
42500	←	425×100	(multiplication by 1 hundred)
57800	←	$425 \times (6 + 30 + 100)$	

Example 5 : The school canteen charges ₹ 20 for lunch and ₹ 4 for milk for each day. How much money do you spend in 5 days on these things?

Solution : This can be found by two methods.

Method 1 : Find the amount for lunch for 5 days.
Find the amount for milk for 5 days.
Then add i.e.
Cost of lunch = 5 × 20 = ₹ 100
Cost of milk = 5 × 4 = ₹ 20
Total cost = ₹ (100 + 20) = ₹ 120



Method 2 : Find the total amount for one day.
Then multiply it by 5 i.e.
Cost of (lunch + milk) for one day = ₹ (20 + 4)
Cost for 5 days = ₹ 5 × (20 + 4) = ₹ (5 × 24)
= ₹ 120.
The example shows that
$5 \times (20 + 4) = (5 \times 20) + (5 \times 4)$
This is the principle of distributivity of multiplication over addition.

Example 6 : Find 12 × 35 using distributivity.

Solution : $12 \times 35 = 12 \times (30 + 5)$
$= 12 \times 30 + 12 \times 5$
$= 360 + 60 = 420$

Try These

Find 15 × 68; 17 × 23; 69 × 78 + 22 × 69 using distributive property.

Example 7 : Simplify: 126 × 55 + 126 × 45

Solution : $126 \times 55 + 126 \times 45 = 126 \times (55 + 45)$
$= 126 \times 100$
$= 12600.$

Identity (for addition and multiplication)

How is the collection of whole numbers different from the collection of natural numbers? It is just the presence of 'zero' in the collection of whole numbers. This number 'zero' has a special role in addition. The following table will help you guess the role.

7	+	0	=	7
5	+	0	=	5
0	+	15	=	15
0	+	26	=	26
0	+	.....	=	.....

When you add zero to any whole number what is the result?

It is the same whole number again! Zero is called an identity for addition of whole numbers or additive identity for whole numbers.

Zero has a special role in multiplication too. Any number when multiplied by zero becomes zero!

For example, observe the pattern :

$5 \times 6 = 30$
$5 \times 5 = 25$
$5 \times 4 = 20$
$5 \times 3 = 15$
$5 \times 2 = ...$
$5 \times 1 = ...$
$5 \times 0 = ?$

Observe how the products decrease.
Do you see a pattern?
Can you guess the last step?
Is this pattern true for other whole numbers also?
Try doing this with two different whole numbers.

You came across an additive identity for whole numbers. A number remains unchanged when added to zero. Similar is the case for a multiplicative identity for whole numbers. Observe this table.

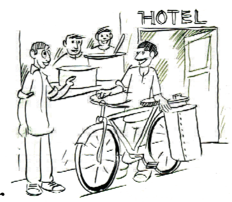
7	×	1	=	7
5	×	1	=	5
1	×	12	=	12
1	×	100	=	100
1	×	......	=	......

You are right. 1 is the identity for multiplication of whole numbers or multiplicative identity for whole numbers.

EXERCISE 2.2

1. Find the sum by suitable rearrangement:
   (a) $837 + 208 + 363$ (b) $1962 + 453 + 1538 + 647$
2. Find the product by suitable rearrangement:
   (a) $2 \times 1768 \times 50$ (b) $4 \times 166 \times 25$ (c) $8 \times 291 \times 125$
   (d) $625 \times 279 \times 16$ (e) $285 \times 5 \times 60$ (f) $125 \times 40 \times 8 \times 25$
3. Find the value of the following:
   (a) $297 \times 17 + 297 \times 3$ (b) $54279 \times 92 + 8 \times 54279$
   (c) $81265 \times 169 – 81265 \times 69$ (d) $3845 \times 5 \times 782 + 769 \times 25 \times 218$
4. Find the product using suitable properties.
   (a) 738×103 (b) 854×102 (c) 258×1008 (d) 1005×168
5. A taxidriver filled his car petrol tank with 40 litres of petrol on Monday. The next day, he filled the tank with 50 litres of petrol. If the petrol costs ₹ 44 per litre, how much did he spend in all on petrol?

6. A vendor supplies 32 litres of milk to a hotel in the morning and 68 litres of milk in the evening. If the milk costs ₹ 45 per litre, how much money is due to the vendor per day?



7. Match the following:

(i)	425 × 136 = 425 × (6 + 30 +100)	(a)	Commutativity under multiplication.
(ii)	2 × 49 × 50 = 2 × 50 × 49	(b)	Commutativity under addition.
(iii)	80 + 2005 + 20 = 80 + 20 + 2005	(c)	Distributivity of multiplication over addition.

2.5 Patterns in Whole Numbers

We shall try to arrange numbers in elementary shapes made up of dots. The shapes we take are (1) a line (2) a rectangle (3) a square and (4) a triangle. Every number should be arranged in one of these shapes. No other shape is allowed.

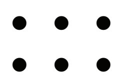
- Every number can be arranged as a line;

  The number 2 is shown as • •
  The number 3 is shown as • • •
  and so on.

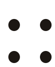
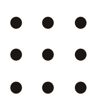
- Some numbers can be shown also as rectangles.

  For example,

  The number 6 can be shown as a rectangle. Note there are 2 rows and 3 columns.

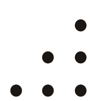


- Some numbers like 4 or 9 can also be arranged as squares;

4 ⟶    9 ⟶ 

- Some numbers can also be arranged as triangles.

  For example,

3 ⟶    6 ⟶ 

Note that the triangle should have its two sides equal. The number of dots in the rows starting from the bottom row should be like 4, 3, 2, 1. The top row should always have 1 dot.

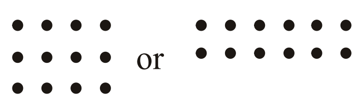
Now, complete the table :

> 1 is a special number.

Number	Line	Rectangle	Square	Triangle
2	Yes	No	No	No
3	Yes	No	No	Yes
4	Yes	Yes	Yes	No
5	Yes	No	No	No
6				
7				
8				
9				
10				
11				
12				
13				

Try These

1. Which numbers can be shown only as a line?
2. Which can be shown as squares?
3. Which can be shown as rectangles?
4. Write down the first seven numbers that can be arranged as triangles, e.g. 3, 6, ...
5. Some numbers can be shown by two rectangles, for example,

12 ⟶ 

3×4 or 2×6

Give at least five other such examples.

Patterns Observation

Observation of patterns can guide you in simplifying processes. Study the following:

(a) $117 + 9 = 117 + 10 - 1 = 127 - 1 = 126$

(b) $117 - 9 = 117 - 10 + 1 = 107 + 1 = 108$

(c) $117 + 99 = 117 + 100 - 1 = 217 - 1 = 216$

(d) $117 - 99 = 117 - 100 + 1 = 17 + 1 = 18$

Does this pattern help you to add or subtract numbers of the form 9, 99, 999,…?

Here is one more pattern :

(a) $84 \times 9 = 84 \times (10 - 1)$ (b) $84 \times 99 = 84 \times (100 - 1)$

(c) $84 \times 999 = 84 \times (1000 - 1)$

Do you find a shortcut to multiply a number by numbers of the form 9, 99, 999,…?

Such shortcuts enable you to do sums verbally.

The following pattern suggests a way of multiplying a number by 5 or 25 or 125. (You can think of extending it further).

(i) $96 \times 5 = 96 \times \frac{10}{2} = \frac{960}{2} = 480$ (ii) $96 \times 25 = 96 \times \frac{100}{4} = \frac{9600}{4} = 2400$

(iii) $96 \times 125 = 96 \times \frac{1000}{8} = \frac{96000}{8} = 12000$...

What does the pattern that follows suggest?

(i) $64 \times 5 = 64 \times \frac{10}{2} = 32 \times 10 = 320 \times 1$

(ii) $64 \times 15 = 64 \times \frac{30}{2} = 32 \times 30 = 320 \times 3$

(iii) $64 \times 25 = 64 \times \frac{50}{2} = 32 \times 50 = 320 \times 5$

(iv) $64 \times 35 = 64 \times \frac{70}{2} = 32 \times 70 = 320 \times 7$.......

EXERCISE 2.3

1. Which of the following will not represent zero:

   (a) $1 + 0$ (b) 0×0 (c) $\frac{0}{2}$ (d) $\frac{10-10}{2}$

2. If the product of two whole numbers is zero, can we say that one or both of them will be zero? Justify through examples.

3. If the product of two whole numbers is 1, can we say that one or both of them will be 1? Justify through examples.
4. Find using distributive property :
   (a) 728×101 (b) 5437×1001 (c) 824×25 (d) 4275×125 (e) 504×35
5. Study the pattern :
   $1 \times 8 + 1 = 9$
   $12 \times 8 + 2 = 98$
   $123 \times 8 + 3 = 987$
   $1234 \times 8 + 4 = 9876$
   $12345 \times 8 + 5 = 98765$
   Write the next two steps. Can you say how the pattern works?
   (Hint: $12345 = 11111 + 1111 + 111 + 11 + 1$).

What have we discussed?

1. The numbers 1, 2, 3,... which we use for counting are known as natural numbers.
2. If you add 1 to a natural number, we get its successor. If you subtract 1 from a natural number, you get its predecessor.
3. Every natural number has a successor. Every natural number except 1 has a predecessor.
4. If we add the number zero to the collection of natural numbers, we get the collection of whole numbers. Thus, the numbers 0, 1, 2, 3,... form the collection of whole numbers.
5. Every whole number has a successor. Every whole number except zero has a predecessor.
6. All natural numbers are whole numbers, but all whole numbers are not natural numbers.
7. We take a line, mark a point on it and label it 0. We then mark out points to the right of 0, at equal intervals. Label them as 1, 2, 3,.... Thus, we have a number line with the whole numbers represented on it. We can easily perform the number operations of addition, subtraction and multiplication on the number line.
8. Addition corresponds to moving to the right on the number line, whereas subtraction corresponds to moving to the left. Multiplication corresponds to making jumps of equal distance starting from zero.
9. Adding two whole numbers always gives a whole number. Similarly, multiplying two whole numbers always gives a whole number. We say that whole numbers are closed under addition and also under multiplication. However, whole numbers are not closed under subtraction and under division.
10. Division by zero is not defined.

11. Zero is the identity for addition of whole numbers. The whole number 1 is the identity for multiplication of whole numbers.
12. You can add two whole numbers in any order. You can multiply two whole numbers in any order. We say that addition and multiplication are commutative for whole numbers.
13. Addition and multiplication, both, are associative for whole numbers.
14. Multiplication is distributive over addition for whole numbers.
15. Commutativity, associativity and distributivity properties of whole numbers are useful in simplifying calculations and we use them without being aware of them.
16. Patterns with numbers are not only interesting, but are useful especially for verbal calculations and help us to understand properties of numbers better.

Playing with Numbers

Chapter 3

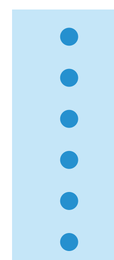
3.1 Introduction

Ramesh has 6 marbles with him. He wants to arrange them in rows in such a way that each row has the same number of marbles. He arranges them in the following ways and matches the total number of marbles.

(i) 1 marble in each row

Number of rows = 6

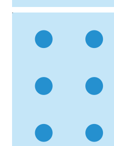
Total number of marbles = $1 \times 6 = 6$



(ii) 2 marbles in each row

Number of rows = 3

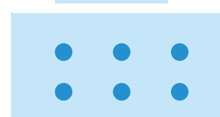
Total number of marbles = $2 \times 3 = 6$



(iii) 3 marbles in each row

Number of rows = 2

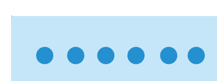
Total number of marbles = $3 \times 2 = 6$



(iv) He could not think of any arrangement in which each row had 4 marbles or 5 marbles. So, the only possible arrangement left was with all the 6 marbles in a row.

Number of rows = 1

Total number of marbles = $6 \times 1 = 6$



From these calculations Ramesh observes that 6 can be written as a product of two numbers in different ways as

$6 = 1 \times 6$; $6 = 2 \times 3$; $6 = 3 \times 2$; $6 = 6 \times 1$

From 6 = 2 × 3 it can be said that 2 and 3 exactly divide 6. So, 2 and 3 are exact divisors of 6. From the other product 6 = 1 × 6, the exact divisors of 6 are found to be 1 and 6.

Thus, 1, 2, 3 and 6 are exact divisors of 6. They are called the **factors** of 6. Try arranging 18 marbles in rows and find the factors of 18.

3.2 Factors and Multiples

Mary wants to find those numbers which exactly divide 4. She divides 4 by numbers less than 4 this way.

1) 4 (4 – 4 0	2) 4 (2 – 4 0	3) 4 (1 – 3 1	4) 4 (1 – 4 0
Quotient is 4 Remainder is 0 4 = 1 × 4	Quotient is 2 Remainder is 0 4 = 2 × 2	Quotient is 1 Remainder is 1	Quotient is 1 Remainder is 0 4 = 4 × 1

She finds that the number 4 can be written as: 4 = 1 × 4; 4 = 2 × 2; 4 = 4 × 1 and knows that the numbers 1, 2 and 4 are exact divisors of 4.

These numbers are called factors of 4.

A factor of a number is an exact divisor of that number.

Observe each of the factors of 4 is less than or equal to 4.

Game-1 : This is a game to be played by two persons say A and B. It is about spotting factors.

It requires 50 pieces of cards numbered 1 to 50.

Arrange the cards on the table like this.

1	2	3	4	5	6	7	
8	9	10	11	12	13	14	
15	16	17	18	19	20	21	
22	23	24	25	26	27	28	
29	30	31	32	33	34	35	
36	37	38	39	40	41	42	
43	44	45	46	47	48	49	50

Steps

(a) Decide who plays first, A or B.

(b) Let A play first. He picks up a card from the table, and keeps it with him. Suppose the card has number 28 on it.

(c) Player B then picks up all those cards having numbers which are factors of the number on A's card (i.e. 28), and puts them in a pile near him.

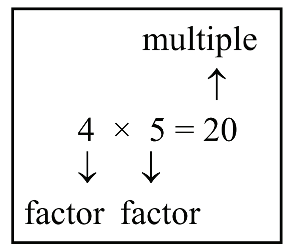
(d) Player B then picks up a card from the table and keeps it with him. From the cards that are left, A picks up all those cards whose numbers are factors of the number on B's card. A puts them on the previous card that he collected.

(e) The game continues like this until all the cards are used up.

(f) A will add up the numbers on the cards that he has collected. B too will do the same with his cards. The player with greater sum will be the winner.

The game can be made more interesting by increasing the number of cards. Play this game with your friend. Can you find some way to win the game?

When we write a number 20 as $20 = 4 \times 5$, we say 4 and 5 are factors of 20. We also say that 20 is a multiple of 4 and 5.



The representation $24 = 2 \times 12$ shows that 2 and 12 are factors of 24, whereas 24 is a multiple of 2 and 12.

We can say that a number is a multiple of each of its factors

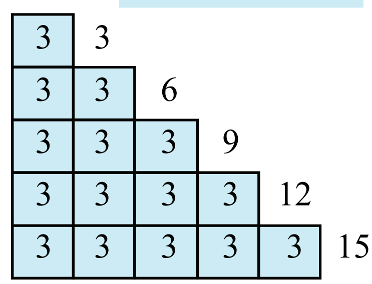
Let us now see some interesting facts about factors and multiples.

Try These

Find the possible factors of 45, 30 and 36.

(a) Collect a number of wooden/paper strips of length 3 units each.

(b) Join them end to end as shown in the following figure.



The length of the strip at the top is $3 = 1 \times 3$ units.

The length of the strip below it is $3 + 3 = 6$ units. Also, $6 = 2 \times 3$. The length of the next strip is $3 + 3 + 3 = 9$ units, and $9 = 3 \times 3$. Continuing this way we can express the other lengths as,

$12 = 4 \times 3$; $\quad 15 = 5 \times 3$

We say that the numbers 3, 6, 9, 12, 15 are multiples of 3.

The list of multiples of 3 can be continued as 18, 21, 24, ...

Each of these multiples is greater than or equal to 3.

The multiples of the number 4 are 4, 8, 12, 16, 20, 24, ...

The list is endless. Each of these numbers is greater than or equal to 4.

Let us see what we conclude about factors and multiples:

1. Is there any number which occurs as a factor of every number ? Yes. It is 1. For example 6 = 1 × 6, 18 = 1 × 18 and so on. Check it for a few more numbers.

   We say **1 is a factor of every number.**

2. Can 7 be a factor of itself ? Yes. You can write 7 as 7 = 7 × 1. What about 10? and 15?.

   You will find that every number can be expressed in this way.

   We say that **every number is a factor of itself.**

3. What are the factors of 16? They are 1, 2, 4, 8, 16. Out of these factors do you find any factor which does not divide 16? Try it for 20; 36.

   You will find that **every factor of a number is an exact divisor of that number.**

4. What are the factors of 34? They are 1, 2, 17 and 34 itself. Out of these which is the greatest factor? It is 34 itself.

   The other factors 1, 2 and 17 are less than 34. Try to check this for 64, 81 and 56.

   We say that **every factor is less than or equal to the given number.**

5. The number 76 has 5 factors. How many factors does 136 or 96 have? You will find that you are able to count the number of factors of each of these.

   Even if the numbers are as large as 10576, 25642 etc. or larger, you can still count the number of factors of such numbers, (though you may find it difficult to factorise such numbers).

   We say that **number of factors of a given number are finite.**

6. What are the multiples of 7? Obviously, 7, 14, 21, 28,... You will find that each of these multiples is greater than or equal to 7. Will it happen with each number? Check this for the multiples of 6, 9 and 10.

   We find that **every multiple of a number is greater than or equal to that number.**

7. Write the multiples of 5. They are 5, 10, 15, 20, ... Do you think this list will end anywhere? No! The list is endless. Try it with multiples of 6,7 etc.

   We find that **the number of multiples of a given number is infinite.**

8. Can 7 be a multiple of itself ? Yes, because 7 = 7×1. Will it be true for other numbers also? Try it with 3, 12 and 16.

   You will find that **every number is a multiple of itself.**

The factors of 6 are 1, 2, 3 and 6. Also, 1+2+3+6 = 12 = 2 × 6. We find that the sum of the factors of 6 is twice the number 6. All the factors of 28 are 1, 2, 4, 7, 14 and 28. Adding these we have, 1 + 2 + 4 + 7 + 14 + 28 = 56 = 2 × 28.

The sum of the factors of 28 is equal to twice the number 28.

A number for which sum of all its factors is equal to twice the number is called a perfect number. The numbers 6 and 28 are perfect numbers.

Is 10 a perfect number?

Example 1 : Write all the factors of 68.

Solution : We note that

68 = 1 × 68   68 = 2 × 34
68 = 4 × 17   68 = 17 × 4

Stop here, because 4 and 17 have occurred earlier.

Thus, all the factors of 68 are 1, 2, 4, 17, 34 and 68.

Example 2 : Find the factors of 36.

Solution : 36 = 1 × 36   36 = 2 × 18   36 = 3 × 12
36 = 4 × 9   36 = 6 × 6

Stop here, because both the factors (6) are same. Thus, the factors are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

Example 3 : Write first five multiples of 6.

Solution : The required multiples are: 6×1= 6, 6×2 = 12, 6×3 = 18, 6×4 = 24, 6×5 = 30 i.e. 6, 12, 18, 24 and 30.

EXERCISE 3.1

1. Write all the factors of the following numbers :
   (a) 24   (b) 15   (c) 21
   (d) 27   (e) 12   (f) 20
   (g) 18   (h) 23   (i) 36
2. Write first five multiples of :
   (a) 5   (b) 8   (c) 9
3. Match the items in column 1 with the items in column 2.

Column 1	**Column 2**
(i) 35	(a) Multiple of 8
(ii) 15	(b) Multiple of 7
(iii) 16	(c) Multiple of 70
(iv) 20	(d) Factor of 30

(v) 25 (e) Factor of 50
(f) Factor of 20

4. Find all the multiples of 9 upto 100.

3.3 Prime and Composite Numbers

We are now familiar with the factors of a number. Observe the number of factors of a few numbers arranged in this table.

Numbers	Factors	Number of Factors
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5	1, 5	2
6	1, 2, 3, 6	4
7	1, 7	2
8	1, 2, 4, 8	4
9	1, 3, 9	3
10	1, 2, 5, 10	4
11	1, 11	2
12	1, 2, 3, 4, 6, 12	6

We find that (a) The number 1 has only one factor (i.e. itself).

(b) There are numbers, having exactly two factors 1 and the number itself. Such number are 2, 3, 5, 7, 11 etc. These numbers are prime numbers.

The numbers other than 1 whose only factors are 1 and the number itself are called Prime numbers.

Try to find some more prime numbers other than these.

(c) There are numbers having more than two factors like 4, 6, 8, 9, 10 and so on. These numbers are composite numbers.

> 1 is neither a prime nor a composite number.

Numbers having more than two factors are called Composite numbers.

Is 15 a composite number? Why? What about 18? 25?

Without actually checking the factors of a number, we can find prime numbers from 1 to 100 with an easier method. This method was given by a

Greek Mathematician **Eratosthenes,** in the third century B.C. Let us see the method. List all numbers from 1 to 100, as shown below.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 1 : Cross out 1 because it is not a prime number.

Step 2 : Encircle 2, cross out all the multiples of 2, other than 2 itself, i.e. 4, 6, 8 and so on.

Step 3 : You will find that the next uncrossed number is 3. Encircle 3 and cross out all the multiples of 3, other than 3 itself.

Step 4 : The next uncrossed number is 5. Encircle 5 and cross out all the multiples of 5 other than 5 itself.

Step 5 : Continue this process till all the numbers in the list are either encircled or crossed out.

All the encircled numbers are prime numbers. All the crossed out numbers, other than 1 are composite numbers.

This method is called the **Sieve of Eratosthenes**.

Try These

Observe that $2 \times 3 + 1 = 7$ is a prime number. Here, 1 has been added to a multiple of 2 to get a prime number. Can you find some more numbers of this type?

Example 4 : Write all the prime numbers less than 15.

Solution : By observing the Sieve Method, we can easily write the required prime numbers as 2, 3, 5, 7, 11 and 13.

even and odd numbers

Do you observe any pattern in the numbers 2, 4, 6, 8, 10, 12, 14, ...? You will find that each of them is a multiple of 2.

These are called *even numbers*. The rest of the numbers 1, 3, 5, 7, 9, 11,... are called *odd numbers*.

You can verify that a two digit number or a three digit number is even or not. How will you know whether a number like 756482 is even? By dividing it by 2. Will it not be tedious?

We say that a number with 0, 2, 4, 6, 8 at the ones place is an *even number.* So, 350, 4862, 59246 are even numbers. The numbers 457, 2359, 8231 are all odd. Let us try to find some interesting facts:

(a) Which is the smallest even number? It is 2. Which is the smallest prime number? It is again 2.

Thus, **2 is the smallest prime number which is even.**

(b) The other prime numbers are 3, 5, 7, 11, 13, ... Do you find any even number in this list? Of course not, they are all odd.

Thus, we can say that **every prime number except 2 is odd.**

EXERCISE 3.2

1. What is the sum of any two (a) Odd numbers? (b) Even numbers?
2. State whether the following statements are True or False:
   (a) The sum of three odd numbers is even.
   (b) The sum of two odd numbers and one even number is even.
   (c) The product of three odd numbers is odd.
   (d) If an even number is divided by 2, the quotient is always odd.
   (e) All prime numbers are odd.
   (f) Prime numbers do not have any factors.
   (g) Sum of two prime numbers is always even.
   (h) 2 is the only even prime number.
   (i) All even numbers are composite numbers.
   (j) The product of two even numbers is always even.
3. The numbers 13 and 31 are prime numbers. Both these numbers have same digits 1 and 3. Find such pairs of prime numbers upto 100.
4. Write down separately the prime and composite numbers less than 20.
5. What is the greatest prime number between 1 and 10?
6. Express the following as the sum of two odd primes.
   (a) 44 (b) 36 (c) 24 (d) 18
7. Give three pairs of prime numbers whose difference is 2.
   [Remark : Two prime numbers whose difference is 2 are called twin primes].
8. Which of the following numbers are prime?
   (a) 23 (b) 51 (c) 37 (d) 26
9. Write seven consecutive composite numbers less than 100 so that there is no prime number between them.

10. Express each of the following numbers as the sum of three odd primes:
    (a) 21 (b) 31 (c) 53 (d) 61
11. Write five pairs of prime numbers less than 20 whose sum is divisible by 5.
    (**Hint :** 3+7 = 10)
12. Fill in the blanks :
    (a) A number which has only two factors is called a ______.
    (b) A number which has more than two factors is called a ______.
    (c) 1 is neither ______ nor ______.
    (d) The smallest prime number is ______.
    (e) The smallest composite number is _____.
    (f) The smallest even number is ______.

3.4 Tests for Divisibility of Numbers

Is the number 38 divisible by 2? by 4? by 5?

By actually dividing 38 by these numbers we find that it is divisible by 2 but not by 4 and by 5.

Let us see whether we can find a pattern that can tell us whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, 10 or 11. Do you think such patterns can be easily seen?

Divisibility by 10 : Charu was looking at the multiples of 10. The multiples are 10, 20, 30, 40, 50, 60, ... She found something common in these numbers. Can you tell what? Each of these numbers has 0 in the ones place.

She thought of some more numbers with 0 at ones place like 100, 1000, 3200, 7010. She also found that all such numbers are divisible by 10.

She finds that **if a number has 0 in the ones place then it is divisible by 10.**

Can you find out the divisibility rule for 100?

Divisibility by 5 : Mani found some interesting pattern in the numbers 5, 10, 15, 20, 25, 30, 35, ... Can you tell the pattern? Look at the units place. All these numbers have either 0 or 5 in their ones place. We know that these numbers are divisible by 5.

Mani took up some more numbers that are divisible by 5, like 105, 215, 6205, 3500. Again these numbers have either 0 or 5 in their ones places.

He tried to divide the numbers 23, 56, 97 by 5. Will he be able to do that? Check it. He observes that **a number which has either 0 or 5 in its ones place is divisible by 5,** other numbers leave a remainder.

Is 1750125 divisible 5?

Divisibility by 2 : Charu observes a few multiples of 2 to be 10, 12, 14, 16... and also numbers like 2410, 4356, 1358, 2972, 5974. She finds some pattern in the ones place of these numbers. Can you tell that? These numbers have only the digits 0, 2, 4, 6, 8 in the ones place.

She divides these numbers by 2 and gets remainder 0.

She also finds that the numbers 2467, 4829 are not divisible by 2. These numbers do not have 0, 2, 4, 6 or 8 in their ones place.

Looking at these observations she concludes that **a number is divisible by 2 if it has any of the digits 0, 2, 4, 6 or 8 in its ones place.**

Divisibility by 3 : Are the numbers 21, 27, 36, 54, 219 divisible by 3? Yes, they are.

Are the numbers 25, 37, 260 divisible by 3? No.

Can you see any pattern in the ones place? We cannot, because numbers with the same digit in the ones places can be divisible by 3, like 27, or may not be divisible by 3 like 17, 37. Let us now try to add the digits of 21, 36, 54 and 219. Do you observe anything special ? 2+1=3, 3+6=9, 5+4=9, 2+1+9=12. All these additions are divisible by 3.

Add the digits in 25, 37, 260. We get 2+5=7, 3+7=10, 2+6+0 = 8.

These are not divisible by 3.

We say that **if the sum of the digits is a multiple of 3, then the number is divisible by 3.**

Is 7221 divisible by 3?

Divisibility by 6 : Can you identify a number which is divisible by both 2 and 3? One such number is 18. Will 18 be divisible by 2×3=6? Yes, it is.

Find some more numbers like 18 and check if they are divisible by 6 also.

Can you quickly think of a number which is divisible by 2 but not by 3?

Now for a number divisible by 3 but not by 2, one example is 27. Is 27 divisible by 6? No. Try to find numbers like 27.

From these observations we conclude that **if a number is divisible by 2 and 3 both then it is divisible by 6 also.**

Divisibility by 4 : Can you quickly give five 3-digit numbers divisible by 4? One such number is 212. Think of such 4-digit numbers. One example is 1936.

Observe the number formed by the ones and tens places of 212. It is 12; which is divisible by 4. For 1936 it is 36, again divisible by 4.

Try the exercise with other such numbers, for example with 4612; 3516; 9532.

Is the number 286 divisible by 4? No. Is 86 divisible by 4? No.

So, we see that **a number with 3 or more digits is divisible by 4 if the**

number formed by its last two digits (i.e. ones and tens) is divisible by 4. Check this rule by taking ten more examples.

Divisibility for 1 or 2 digit numbers by 4 has to be checked by actual division.

Divisibility by 8 : Are the numbers 1000, 2104, 1416 divisible by 8?

You can check that they are divisible by 8. Let us try to see the pattern.

Look at the digits at ones, tens and hundreds place of these numbers. These are 000, 104 and 416 respectively. These too are divisible by 8. Find some more numbers in which the number formed by the digits at units, tens and hundreds place (i.e. last 3 digits) is divisible by 8. For example, 9216, 8216, 7216, 10216, 9995216 etc. You will find that the numbers themselves are divisible by 8.

We find that **a number with 4 or more digits is divisible by 8, if the number formed by the last three digits is divisible by 8.**

Is 73512 divisible by 8?

The divisibility for numbers with 1, 2 or 3 digits by 8 has to be checked by actual division.

Divisibility by 9 : The multiples of 9 are 9, 18, 27, 36, 45, 54,... There are other numbers like 4608, 5283 that are also divisible by 9.

Do you find any pattern when the digits of these numbers are added?

1 + 8 = 9, 2 + 7 = 9, 3 + 6 = 9, 4 + 5 = 9

4 + 6 + 0 + 8 = 18, 5 + 2 + 8 + 3 = 18

All these sums are also divisible by 9.

Is the number 758 divisible by 9?

No. The sum of its digits 7 + 5 + 8 = 20 is also not divisible by 9.

These observations lead us to say that **if the sum of the digits of a number is divisible by 9, then the number itself is divisible by 9.**

Divisibility by 11 : The numbers 308, 1331 and 61809 are all divisible by 11. We form a table and see if the digits in these numbers lead us to some pattern.

Number	Sum of the digits (at odd places) from the right	Sum of the digits (at even places) from the right	Difference
308	8 + 3 = 11	0	11 – 0 = 11
1331	1 + 3 = 4	3 + 1 = 4	4 – 4 = 0
61809	9 + 8 + 6 = 23	0 + 1 = 1	23 – 1 = 22

We observe that in each case the difference is either 0 or divisible by 11. All these numbers are also divisible by 11.

For the number 5081, the difference of the digits is (5+8) – (1+0) = 12 which is not divisible by 11. The number 5081 is also not divisible by 11.

Thus, to check the divisibility of a number by 11, the rule is, **find the difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) of the number. If the difference is either 0 or divisible by 11, then the number is divisible by 11**.

EXERCISE 3.3

1. Using divisibility tests, determine which of the following numbers are divisible by 2; by 3; by 4; by 5; by 6; by 8; by 9; by 10 ; by 11 (say, yes or no):

Number	Divisible by								
	2	3	4	5	6	8	9	10	11
128	Yes	No	Yes	No	No	Yes	No	No	No
990	.....	.....	.....	.....	.....	.....	.....	.....	.....
1586	.....	.....	.....	.....	.....	.....	.....	.....	.....
275	.....	.....	.....	.....	.....	.....	.....	.....	.....
6686	.....	.....	.....	.....	.....	.....	.....	.....	.....
639210	.....	.....	.....	.....	.....	.....	.....	.....	.....
429714	.....	.....	.....	.....	.....	.....	.....	.....	.....
2856	.....	.....	.....	.....	.....	.....	.....	.....	.....
3060	.....	.....	.....	.....	.....	.....	.....	.....	.....
406839	.....	.....	.....	.....	.....	.....	.....	.....	.....

2. Using divisibility tests, determine which of the following numbers are divisible by 4; by 8:
   (a) 572 (b) 726352 (c) 5500 (d) 6000 (e) 12159
   (f) 14560 (g) 21084 (h) 31795072 (i) 1700 (j) 2150
3. Using divisibility tests, determine which of following numbers are divisible by 6:
   (a) 297144 (b) 1258 (c) 4335 (d) 61233 (e) 901352
   (f) 438750 (g) 1790184 (h) 12583 (i) 639210 (j) 17852
4. Using divisibility tests, determine which of the following numbers are divisible by 11:
   (a) 5445 (b) 10824 (c) 7138965 (d) 70169308 (e) 10000001
   (f) 901153
5. Write the smallest digit and the greatest digit in the blank space of each of the following numbers so that the number formed is divisible by 3 :
   (a) __ 6724 (b) 4765 __ 2

6. Write a digit in the blank space of each of the following numbers so that the number formed is divisible by 11 :
   (a) 92 __ 389 (b) 8 __ 9484

3.5 Common Factors and Common Multiples

Observe the factors of some numbers taken in pairs.

Try These

Find the common factors of
(a) 8, 20 (b) 9, 15

(a) What are the factors of 4 and 18?

The factors of 4 are 1, 2 and 4.

The factors of 18 are 1, 2, 3, 6, 9 and 18.

The numbers 1 and 2 are the factors of both 4 and 18.

They are the common factors of 4 and 18.

(b) What are the common factors of 4 and 15?

These two numbers have only 1 as the common factor.

What about 7 and 16?

Two numbers having only 1 as a common factor are called co-prime numbers. Thus, 4 and 15 are co-prime numbers.

Are 7 and 15, 12 and 49, 18 and 23 co-prime numbers?

(c) Can we find the common factors of 4, 12 and 16?
Factors of 4 are 1, 2 and 4.
Factors of 12 are 1, 2, 3, 4, 6 and 12.
Factors of 16 are 1, 2, 4, 8 and 16.
Clearly, 1, 2 and 4 are the common factors of 4, 12, and 16.
Find the common factors of (a) 8, 12, 20 (b) 9, 15, 21.

Let us now look at the multiples of more than one number taken at a time.

(a) What are the multiples of 4 and 6?

The multiples of 4 are 4, 8, 12, 16, 20, 24, ... (write a few more)

The multiples of 6 are 6, 12, 18, 24, 30, 36, ... (write a few more)

Out of these, are there any numbers which occur in both the lists?

We observe that 12, 24, 36, ... are multiples of both 4 and 6.
Can you write a few more?
They are called the common multiples of 4 and 6.

(b) Find the common multiples of 3, 5 and 6.
Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, ...
Multiples of 5 are 5, 10, 15, 20, 25, 30, 35, ...
Multiples of 6 are 6, 12, 18, 24, 30, ...
Common multiples of 3, 5 and 6 are 30, 60, ...

Write a few more common multiples of 3, 5 and 6.

Example 5 : Find the common factors of 75, 60 and 210.

Solution : Factors of 75 are 1, 3, 5, 15, 25 and 75.

Factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 30 and 60.

Factors of 210 are 1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105 and 210.

Thus, common factors of 75, 60 and 210 are 1, 3, 5 and 15.

Example 6 : Find the common multiples of 3, 4 and 9.

Solution : Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, ...

Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48,...

Multiples of 9 are 9, 18, 27, 36, 45, 54, 63, 72, 81, ...

Clearly, common multiples of 3, 4 and 9 are 36, 72, 108,...

EXERCISE 3.4

1. Find the common factors of :
   (a) 20 and 28 (b) 15 and 25 (c) 35 and 50 (d) 56 and 120
2. Find the common factors of :
   (a) 4, 8 and 12 (b) 5, 15 and 25
3. Find first three common multiples of :
   (a) 6 and 8 (b) 12 and 18
4. Write all the numbers less than 100 which are common multiples of 3 and 4.
5. Which of the following numbers are co-prime?
   (a) 18 and 35 (b) 15 and 37 (c) 30 and 415
   (d) 17 and 68 (e) 216 and 215 (f) 81 and 16
6. A number is divisible by both 5 and 12. By which other number will that number be always divisible?
7. A number is divisible by 12. By what other numbers will that number be divisible?

3.6 Some More Divisibility Rules

Let us observe a few more rules about the divisibility of numbers.

(i) Can you give a factor of 18? It is 9. Name a factor of 9? It is 3. Is 3 a factor of 18? Yes it is. Take any other factor of 18, say 6. Now, 2 is a factor of 6 and it also divides 18. Check this for the other factors of 18. Consider 24. It is divisible by 8 and the factors of 8 i.e. 1, 2, 4 and 8 also divide 24.

So, we may say that **if a number is divisible by another number then it is divisible by each of the factors of that number.**

(ii) The number 80 is divisible by 4 and 5. It is also divisible by $4 \times 5 = 20$, and 4 and 5 are co-primes.
Similarly, 60 is divisible by 3 and 5 which are co-primes. 60 is also divisible by $3 \times 5 = 15$.
If a number is divisible by two co-prime numbers then it is divisible by their product also.

(iii) The numbers 16 and 20 are both divisible by 4. The number $16 + 20 = 36$ is also divisible by 4. Check this for other pairs of numbers.
Try this for other common divisors of 16 and 20.
If two given numbers are divisible by a number, then their sum is also divisible by that number.

(iv) The numbers 35 and 20 are both divisible by 5. Is their difference $35 - 20 = 15$ also divisible by 5 ? Try this for other pairs of numbers also.
If two given numbers are divisible by a number, then their difference is also divisible by that number.
Take different pairs of numbers and check the four rules given above.

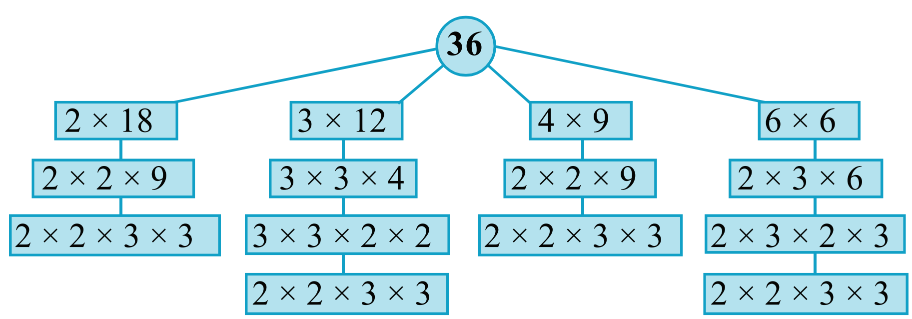
3.7 Prime Factorisation

When a number is expressed as a product of its factors we say that the number has been factorised. Thus, when we write $24 = 3 \times 8$, we say that 24 has been factorised. This is one of the factorisations of 24. The others are :

$24 = 2 \times 12$	$24 = 4 \times 6$	$24 = 3 \times 8$
$= 2 \times 2 \times 6$	$= 2 \times 2 \times 6$	$= 3 \times 2 \times 2 \times 2$
$= 2 \times 2 \times 2 \times 3$	$= 2 \times 2 \times 2 \times 3$	$= 2 \times 2 \times 2 \times 3$

In all the above factorisations of 24, we ultimately arrive at only one factorisation $2 \times 2 \times 2 \times 3$. In this factorisation the only factors 2 and 3 are prime numbers. Such a factorisation of a number is called a *prime factorisation*.

Let us check this for the number 36.



The prime factorisation of 36 is $2 \times 2 \times 3 \times 3$. i.e. the only prime factorisation of 36.

Do This

Try These

Write the prime factorisations of 16, 28, 38.

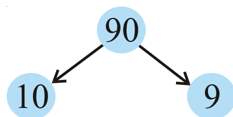
Factor tree

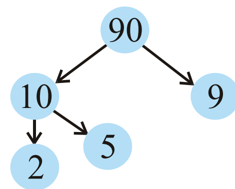
Choose a number and write it

Think of a factor pair say, 90=10×9
90

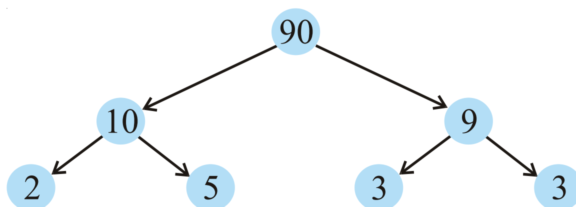
Now think of a factor pair of 10
10 = 2×5

Write factor pair of 9
9 = 3 × 3





Try this for the numbers
(a) 8 (b) 12



Example 7 : Find the prime factorisation of 980.

Solution : We proceed as follows:

We divide the number 980 by 2, 3, 5, 7 etc. in this order repeatedly so long as the quotient is divisible by that number. Thus, the prime factorisation of 980 is 2 × 2 × 5 × 7 × 7.

2	980
2	490
5	245
7	49
7	7
	1

EXERCISE 3.5

1. Which of the following statements are true?
   (a) If a number is divisible by 3, it must be divisible by 9.
   (b) If a number is divisible by 9, it must be divisible by 3.
   (c) A number is divisible by 18, if it is divisible by both 3 and 6.
   (d) If a number is divisible by 9 and 10 both, then it must be divisible by 90.
   (e) If two numbers are co-primes, at least one of them must be prime.
   (f) All numbers which are divisible by 4 must also be divisible by 8.

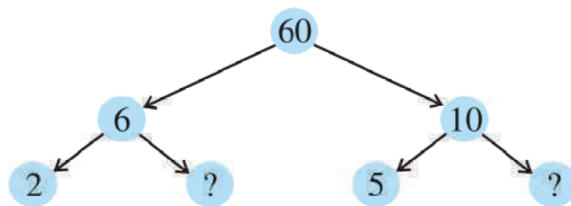
(g) All numbers which are divisible by 8 must also be divisible by 4.

(h) If a number exactly divides two numbers separately, it must exactly divide their sum.

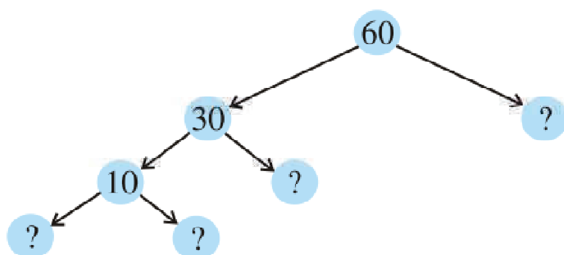
(i) If a number exactly divides the sum of two numbers, it must exactly divide the two numbers separately.

2. Here are two different factor trees for 60. Write the missing numbers.

(a)



(b)



3. Which factors are not included in the prime factorisation of a composite number?
4. Write the greatest 4-digit number and express it in terms of its prime factors.
5. Write the smallest 5-digit number and express it in the form of its prime factors.
6. Find all the prime factors of 1729 and arrange them in ascending order. Now state the relation, if any; between two consecutive prime factors.
7. The product of three consecutive numbers is always divisible by 6. Verify this statement with the help of some examples.
8. The sum of two consecutive odd numbers is divisible by 4. Verify this statement with the help of some examples.
9. In which of the following expressions, prime factorisation has been done?

   (a) $24 = 2 \times 3 \times 4$ (b) $56 = 7 \times 2 \times 2 \times 2$

   (c) $70 = 2 \times 5 \times 7$ (d) $54 = 2 \times 3 \times 9$
10. Determine if 25110 is divisible by 45.

    [**Hint :** 5 and 9 are co-prime numbers. Test the divisibility of the number by 5 and 9].
11. 18 is divisible by both 2 and 3. It is also divisible by $2 \times 3 = 6$. Similarly, a number is divisible by both 4 and 6. Can we say that the number must also be divisible by $4 \times 6 = 24$? If not, give an example to justify your answer.
12. I am the smallest number, having four different prime factors. Can you find me?

3.8 Highest Common Factor

We can find the common factors of any two numbers. We now try to find the highest of these common factors.

What are the common factors of 12 and 16? They are 1, 2 and 4.

What is the highest of these common factors? It is 4.

What are the common factors of 20, 28 and 36? They are 1, 2 and 4 and again 4 is highest of these common factors.

Try These

Find the HCF of the following:

(i) 24 and 36 (ii) 15, 25 and 30

(iii) 8 and 12 (iv) 12, 16 and 28

The Highest Common Factor (HCF) of two or more given numbers is the highest (or greatest) of their common factors. It is also known as Greatest Common Divisor (GCD).

The HCF of 20, 28 and 36 can also be found by prime factorisation of these numbers as follows:

2	20
2	10
5	5
	1

2	28
2	14
7	7
	1

2	36
2	18
3	9
3	3
	1

Thus,

$$20 = \boxed{2} \times \boxed{2} \times 5$$
$$28 = \boxed{2} \times \boxed{2} \times 7$$
$$36 = \boxed{2} \times \boxed{2} \times 3 \times 3$$

The common factor of 20, 28 and 36 is 2(occuring twice). Thus, HCF of 20, 28 and 36 is 2 × 2 = 4.

EXERCISE 3.6

1. Find the HCF of the following numbers :
   (a) 18, 48 (b) 30, 42 (c) 18, 60 (d) 27, 63
   (e) 36, 84 (f) 34, 102 (g) 70, 105, 175
   (h) 91, 112, 49 (i) 18, 54, 81 (j) 12, 45, 75
2. What is the HCF of two consecutive
   (a) numbers? (b) even numbers? (c) odd numbers?

3. HCF of co-prime numbers 4 and 15 was found as follows by factorisation :
   $4 = 2 \times 2$ and $15 = 3 \times 5$ since there is no common prime factor, so HCF of 4 and 15 is 0. Is the answer correct? If not, what is the correct HCF?

3.9 Lowest Common Multiple

What are the common multiples of 4 and 6? They are 12, 24, 36, ... Which is the lowest of these? It is 12. We say that lowest common multiple of 4 and 6 is 12. It is the smallest number that both the numbers are factors of this number.

The Lowest Common Multiple (LCM) of two or more given numbers is the lowest (or smallest or least) of their common multiples.

What will be the LCM of 8 and 12? 4 and 9? 6 and 9?

Example 8 : Find the LCM of 12 and 18.

Solution : We know that common multiples of 12 and 18 are 36, 72, 108 etc. The lowest of these is 36. Let us see another method to find LCM of two numbers.

The prime factorisations of 12 and 18 are :

$12 = 2 \times 2 \times 3; \quad 18 = 2 \times 3 \times 3$

In these prime factorisations, the maximum number of times the prime factor 2 occurs is two; this happens for 12. Similarly, the maximum number of times the factor 3 occurs is two; this happens for 18. The LCM of the two numbers is the product of the prime factors counted the maximum number of times they occur in any of the numbers. Thus, in this case LCM $= 2 \times 2 \times 3 \times 3 = 36$.

Example 9 : Find the LCM of 24 and 90.

Solution : The prime factorisations of 24 and 90 are:

$24 = 2 \times 2 \times 2 \times 3; \quad 90 = 2 \times 3 \times 3 \times 5$

In these prime factorisations the maximum number of times the prime factor 2 occurs is three; this happens for 24. Similarly, the maximum number of times the prime factor 3 occurs is two; this happens for 90. The prime factor 5 occurs only once in 90.

Thus, LCM $= (2 \times 2 \times 2) \times (3 \times 3) \times 5 = 360$

Example 10 : Find the LCM of 40, 48 and 45.

Solution : The prime factorisations of 40, 48 and 45 are;

$40 = 2 \times 2 \times 2 \times 5$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

$45 = 3 \times 3 \times 5$

The prime factor 2 appears maximum number of four times in the prime factorisation of 48, the prime factor 3 occurs maximum number of two times in the prime factorisation of 45, The prime factor 5 appears one time in the prime factorisations of 40 and 45, we take it only once.

Therefore, required LCM = $(2 \times 2 \times 2 \times 2) \times (3 \times 3) \times 5 = 720$

LCM can also be found in the following way :

Example 11 : Find the LCM of 20, 25 and 30.

Solution : We write the numbers as follows in a row :

2	20	25	30	**(A)**
2	10	25	15	**(B)**
3	5	25	15	**(C)**
5	5	25	5	**(D)**
5	1	5	1	**(E)**
	1	1	1	

So, LCM = $2 \times 2 \times 3 \times 5 \times 5$.

(A) Divide by the least prime number which divides atleast one of the given numbers. Here, it is 2. The numbers like 25 are not divisible by 2 so they are written as such in the next row.

(B) Again divide by 2. Continue this till we have no multiples of 2.

(C) Divide by next prime number which is 3.

(D) Divide by next prime number which is 5.

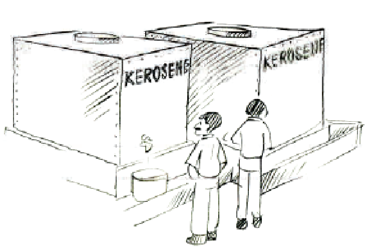
(E) Again divide by 5.

3.10 Some Problems on HCF and LCM

We come across a number of situations in which we make use of the concepts of HCF and LCM. We explain these situations through a few examples.

Example 12 : Two tankers contain 850 litres and 680 litres of kerosene oil respectively. Find the maximum capacity of a container which can measure the kerosene oil of both the tankers when used an exact number of times.

Solution : The required container has to measure both the tankers in a way that the count is an exact number of times. So its capacity must be an exact divisor of the capacities of both the tankers. Moreover, this capacity should be **maximum**. Thus, the maximum capacity of such a container will be the HCF of 850 and 680.



It is found as follows :

2	850
5	425
5	85
17	17
	1

2	680
2	340
2	170
5	85
17	17
	1

Hence,

$850 = 2 \times 5 \times 5 \times 17 \quad = \boxed{2} \times \boxed{5} \times \boxed{17} \times 5$ and

$680 = 2 \times 2 \times 2 \times 5 \times 17 \quad = \boxed{2} \times \boxed{5} \times \boxed{17} \times 2 \times 2$

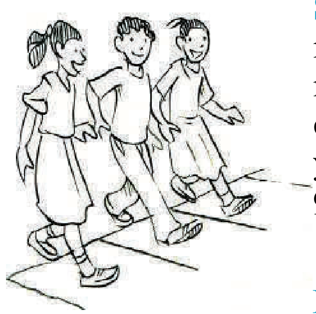
The common factors of 850 and 680 are 2, 5 and 17.

Thus, the HCF of 850 and 680 is $2 \times 5 \times 17 = 170$.

Therefore, maximum capacity of the required container is 170 litres.

It will fill the first container in 5 and the second in 4 refills.

Example 13 : In a morning walk, three persons step off together. Their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?



Solution : The distance covered by each one of them is required to be the same as well as minimum. The required minimum distance each should walk would be the lowest common multiple of the measures of their steps. Can you describe why? Thus, we find the LCM of 80, 85 and 90. The LCM of 80, 85 and 90 is 12240.

The required minimum distance is 12240 cm.

Example 14 : Find the least number which when divided by 12, 16, 24 and 36 leaves a remainder 7 in each case.

Solution : We first find the LCM of 12, 16, 24 and 36 as follows :

2	12	16	24	36
2	6	8	12	18
2	3	4	6	9
2	3	2	3	9
3	3	1	3	9
3	1	1	1	3
	1	1	1	1

Thus, LCM = $2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$

144 is the least number which when divided by the given numbers will leave remainder 0 in each case. But we need the least number that leaves remainder 7 in each case.

Therefore, the required number is 7 more than 144. The required least number = 144 + 7 = 151.

EXERCISE 3.7

1. Renu purchases two bags of fertiliser of weights 75 kg and 69 kg. Find the maximum value of weight which can measure the weight of the fertiliser exact number of times.
2. Three boys step off together from the same spot. Their steps measure 63 cm, 70 cm and 77 cm respectively. What is the minimum distance each should cover so that all can cover the distance in complete steps?
3. The length, breadth and height of a room are 825 cm, 675 cm and 450 cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.
4. Determine the smallest 3-digit number which is exactly divisible by 6, 8 and 12.
5. Determine the greatest 3-digit number exactly divisible by 8, 10 and 12.
6. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change simultaneously again?
7. Three tankers contain 403 litres, 434 litres and 465 litres of diesel respectively. Find the maximum capacity of a container that can measure the diesel of the three containers exact number of times.
8. Find the least number which when divided by 6, 15 and 18 leave remainder 5 in each case.
9. Find the smallest 4-digit number which is divisible by 18, 24 and 32.
10. Find the LCM of the following numbers :

    (a) 9 and 4 (b) 12 and 5 (c) 6 and 5 (d) 15 and 4

    Observe a common property in the obtained LCMs. Is LCM the product of two numbers in each case?
11. Find the LCM of the following numbers in which one number is the factor of the other.

    (a) 5, 20 (b) 6, 18 (c) 12, 48 (d) 9, 45

    What do you observe in the results obtained?

What have we discussed?

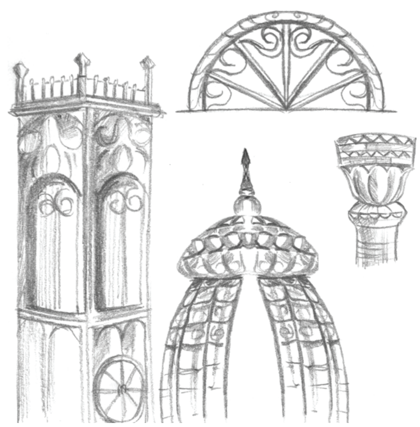
1. We have discussed multiples, divisors, factors and have seen how to identify factors and multiples.
2. We have discussed and discovered the following :
   (a) A factor of a number is an exact divisor of that number.
   (b) Every number is a factor of itself. 1 is a factor of every number.
   (c) Every factor of a number is less than or equal to the given number.
   (d) Every number is a multiple of each of its factors.
   (e) Every multiple of a given number is greater than or equal to that number.
   (f) Every number is a multiple of itself.
3. We have learnt that –
   (a) The number other than 1, with only factors namely 1 and the number itself, is a prime number. Numbers that have more than two factors are called composite numbers. Number 1 is neither prime nor composite.
   (b) The number 2 is the smallest prime number and is even. Every prime number other than 2 is odd.
   (c) Two numbers with only 1 as a common factor are called co-prime numbers.
   (d) If a number is divisible by another number then it is divisible by each of the factors of that number.
   (e) A number divisible by two co-prime numbers is divisible by their product also.
4. We have discussed how we can find just by looking at a number, whether it is divisible by small numbers 2,3,4,5,8,9 and 11. We have explored the relationship between digits of the numbers and their divisibility by different numbers.
   (a) Divisibility by 2,5 and 10 can be seen by just the last digit.
   (b) Divisibility by 3 and 9 is checked by finding the sum of all digits.
   (c) Divisibility by 4 and 8 is checked by the last 2 and 3 digits respectively.
   (d) Divisibility of 11 is checked by comparing the sum of digits at odd and even places.
5. We have discovered that if two numbers are divisible by a number then their sum and difference are also divisible by that number.
6. We have learnt that –
   (a) The Highest Common Factor (HCF) of two or more given numbers is the highest of their common factors.
   (b) The Lowest Common Multiple (LCM) of two or more given numbers is the lowest of their common multiples.

Basic Geometrical Ideas

Chapter 4

4.1 Introduction

Geometry has a long and rich history. The term 'Geometry' is the English equivalent of the Greek word '*Geometron*'. '*Geo*' means Earth and '*metron*' means Measurement. According to historians, the geometrical ideas shaped up in ancient times, probably due to the need in art, architecture and measurement. These include occasions when the boundaries of cultivated lands had to be marked without giving room for complaints. Construction of magnificent palaces, temples, lakes, dams and cities, art and architecture propped up these ideas. Even today geometrical ideas are reflected in all forms of art, measurements, architecture, engineering, cloth designing etc. You observe and use different objects like boxes, tables, books, the tiffin box you carry to your school for lunch, the ball with which you play and so on. All such objects have different shapes. The ruler which you use, the pencil with which you write are straight. The pictures of a bangle, the one rupee coin or a ball appear round.



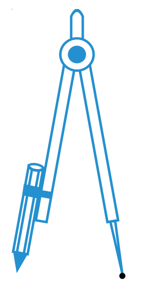
Here, you will learn some interesting facts that will help you know more about the shapes around you.

4.2 Points

By a sharp tip of the pencil, mark a dot on the paper. Sharper the tip, thinner will be the dot. This almost invisible tiny dot will give you an idea of a point.

A point determines a location.

These are some models for a point :


The tip of a compass


The sharpened end of a pencil


The pointed end of a needle

If you mark three points on a paper, you would be required to distinguish them. For this they are denoted by a single capital letter like A,B,C.

•B

•A

•C

These points will be read as point A, point B and point C.

Of course, the dots have to be invisibly thin.

Try These

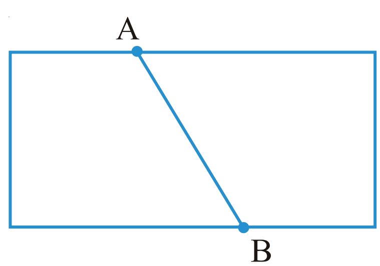
1. With a sharp tip of the pencil, mark four points on a paper and name them by the letters A,C,P,H. Try to name these points in different ways. One such way could be this

   A• •C

   P• •H

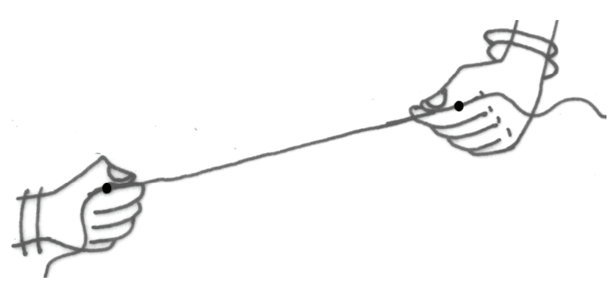
2. A star in the sky also gives us an idea of a point. Identify at least five such situations in your daily life.

4.3 A Line Segment

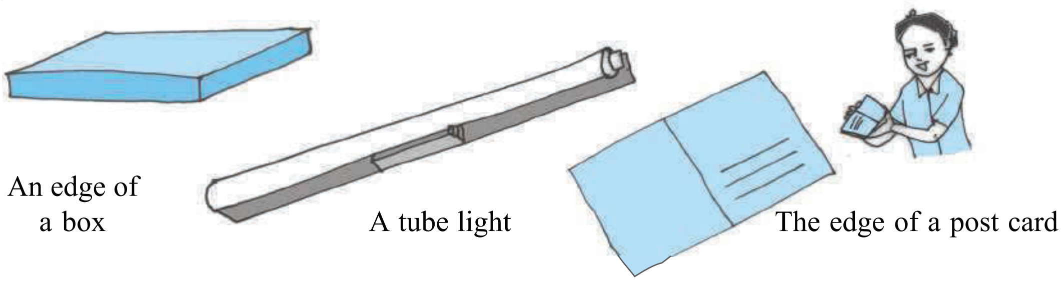


Fold a piece of paper and unfold it. Do you see a fold? This gives the idea of a line segment. It has two end points A and B.

Take a thin thread. Hold its two ends and stretch it without a slack. It represents a line segment. The ends held by hands are the end points of the line segment.



The following are some models for a line segment :



Try to find more examples for line segments from your surroundings.

Mark any two points A and B on a sheet of paper. Try to connect A to B by all possible routes. (Fig 4.1)

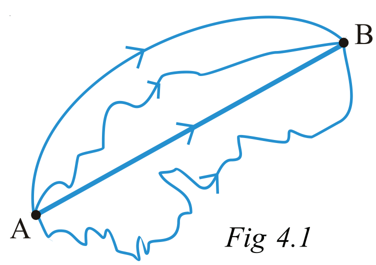


Fig 4.1

What is the shortest route from A to B?

This shortest join of point A to B (including A and B) shown here is a line segment. It is denoted by $\overline{AB}$ or $\overline{BA}$. The points A and B are called the end points of the segment.

Try These

1. Name the line segments in the figure 4.2.
   Is A, the end point of each line segment?

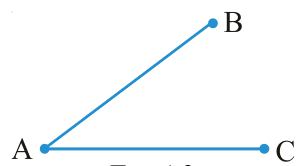


Fig 4.2

4.4 A Line

Imagine that the line segment from A to B (i.e. $\overline{AB}$) is extended beyond A in one direction and beyond B in the other direction without any end (see figure). You now get a **model for a line**.

Do you think you can draw a complete picture of a line? No. (Why?)

A line through two points A and B is written as $\overleftrightarrow{AB}$. It extends indefinitely in both directions. So it contains a countless number of points. (Think about this).

Two points are enough to fix a line. We say 'two points determine a line'.

The adjacent diagram (Fig 4.3) is that of a line PQ written as $\overleftrightarrow{PQ}$. Sometimes a line is denoted by a letter like *l*, *m*.

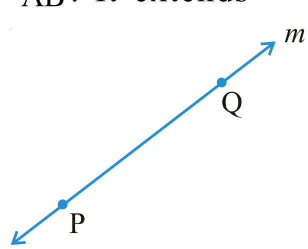


Fig 4.3

4.5 Intersecting Lines

Look at the diagram (Fig 4.4). Two lines l_1 and l_2 are shown. Both the lines pass through point P. We say l_1 and l_2 intersect at P. If two lines have one common point, they are called *intersecting lines*.

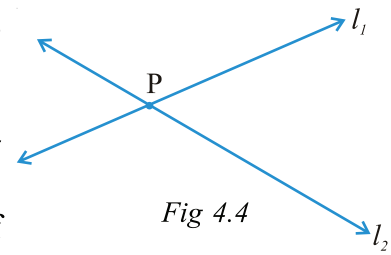
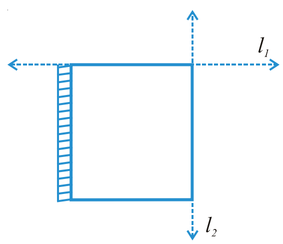


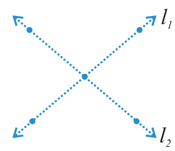
Fig 4.4

The following are some models of a pair of intersecting lines (Fig 4.5) :

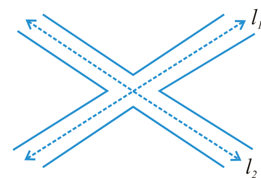
Try to find out some more models for a pair of intersecting lines.



Two adjacement edges of your notebook



The letter X of the English alphabet



Crossing-roads

Fig 4.5

Do This

Take a sheet of paper. Make two folds (and crease them) to represent a pair of intersecting lines and discuss :

(a) Can two lines intersect in more than one point?

(b) Can more than two lines intersect in one point?

4.6 Parallel Lines

Let us look at this table (Fig 4.6). The top ABCD is flat. Are you able to see some points and line segments?

Are there intersecting line segments?

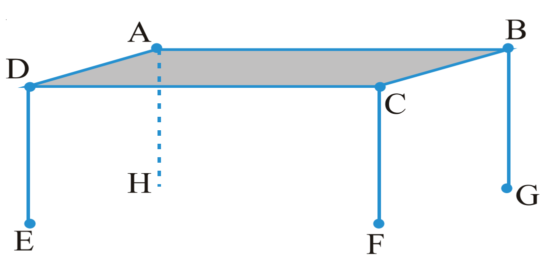


Fig 4.6

Yes, $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ intersect at the point B.

Which line segments intersect at A? at C? at D?

Do the lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{CD}$ intersect?

Do the lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ intersect?

You find that on the table's surface there are line segment which will not meet, however far they are extended. $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ form one such pair. Can you identify one more such pair of lines (which do not meet) on the top of the table?

Lines like these which do not meet are said to be parallel; and are called **parallel lines**.

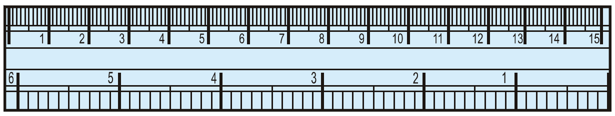
Think, discuss and write

Where else do you see parallel lines? Try to find ten examples.

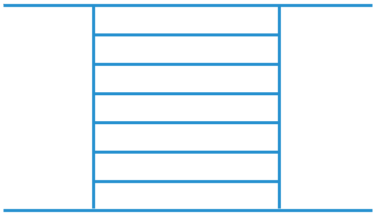
If two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, we write $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

If two lines l_1 and l_2 are parallel, we write $l_1 \parallel l_2$.

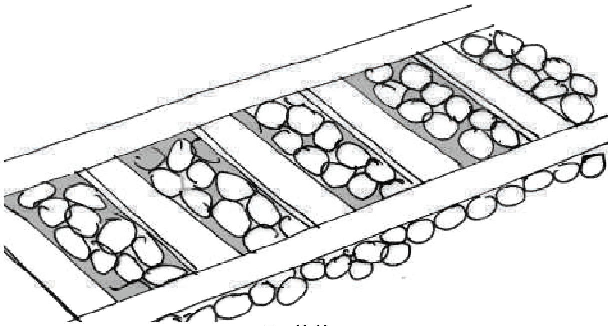
Can you identify parrallel lines in the following figures?



The opposite edges of ruler (scale)

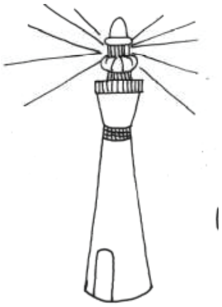


The cross-bars of this window

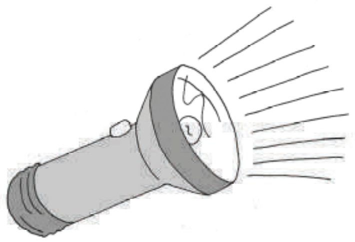


Rail lines

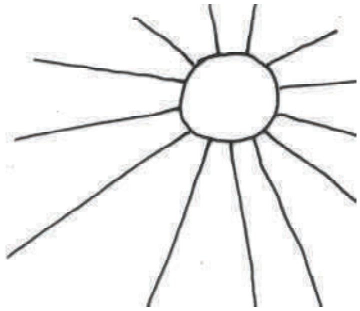
4.7 Ray



Beam of light from a light house



Ray of light from a torch



Sun rays

The following are some models for a ray :

A *ray* is a portion of a line. It starts at one point (called starting point or initial point) and goes endlessly in a direction.

Look at the diagram (Fig 4.7) of ray shown here. Two points are shown on the ray. They are (a) A, the starting point (b) P, a point on the path of the ray.

We denote it by $\overrightarrow{AP}$.

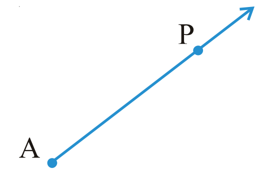


Fig 4.7

Think, discuss and write

If $\overrightarrow{PQ}$ is a ray,

(a) What is its starting point?

(b) Where does the point Q lie on the ray?

(c) Can we say that Q is the starting point of this ray?

Try These

1. Name the rays given in this picture (Fig 4.8).
2. Is T a starting point of each of these rays?

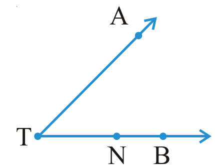


Fig 4.8

Here is a ray $\overrightarrow{OA}$ (Fig 4.9). It starts at O and passes through the point A. It also passes through the point B.

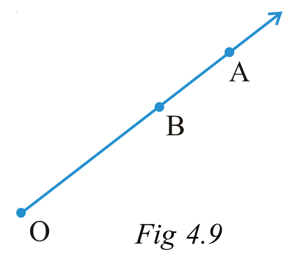


Fig 4.9

Can you also name it as $\overrightarrow{OB}$? Why?

$\overrightarrow{OA}$ and $\overrightarrow{OB}$ are same here.

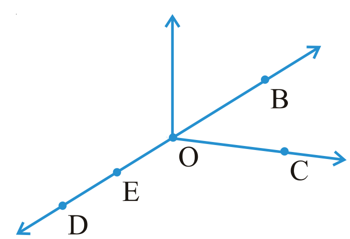
Can we write $\overrightarrow{OA}$ as $\overrightarrow{AO}$? Why or why not?

Draw five rays and write appropriate names for them.

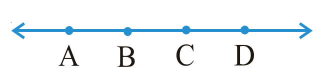
What do the arrows on each of these rays show?

EXERCISE 4.1

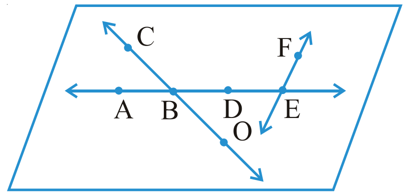
1. Use the figure to name :
   (a) Five points
   (b) A line
   (c) Four rays
   (d) Five line segments



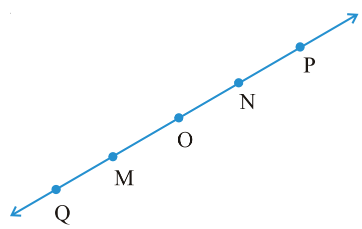
2. Name the line given in all possible (twelve) ways, choosing only two letters at a time from the four given.



3. Use the figure to name :
   (a) Line containing point E.
   (b) Line passing through A.
   (c) Line on which O lies
   (d) Two pairs of intersecting lines.



4. How many lines can pass through (a) one given point? (b) two given points?
5. Draw a rough figure and label suitably in each of the following cases:
   (a) Point P lies on $\overline{AB}$.
   (b) $\overleftrightarrow{XY}$ and $\overleftrightarrow{PQ}$ intersect at M.
   (c) Line l contains E and F but not D.
   (d) $\overleftrightarrow{OP}$ and $\overleftrightarrow{OQ}$ meet at O.
6. Consider the following figure of line $\overleftrightarrow{MN}$. Say whether following statements are true or false in context of the given figure.
   (a) Q, M, O, N, P are points on the line $\overleftrightarrow{MN}$.
   (b) M, O, N are points on a line segment $\overline{MN}$.
   (c) M and N are end points of line segment $\overline{MN}$.
   (d) O and N are end points of line segment $\overline{OP}$.
   (e) M is one of the end points of line segment $\overline{QO}$.
   (f) M is point on ray $\overrightarrow{OP}$.
   (g) Ray $\overrightarrow{OP}$ is different from ray $\overrightarrow{QP}$.
   (h) Ray $\overrightarrow{OP}$ is same as ray $\overrightarrow{OM}$.
   (i) Ray $\overrightarrow{OM}$ is not opposite to ray $\overrightarrow{OP}$.
   (j) O is not an initial point of $\overrightarrow{OP}$.
   (k) N is the initial point of $\overrightarrow{NP}$ and $\overrightarrow{NM}$.



4.8 Curves

Have you ever taken a piece of paper and just doodled? The pictures that are results of your doodling are called *curves*.

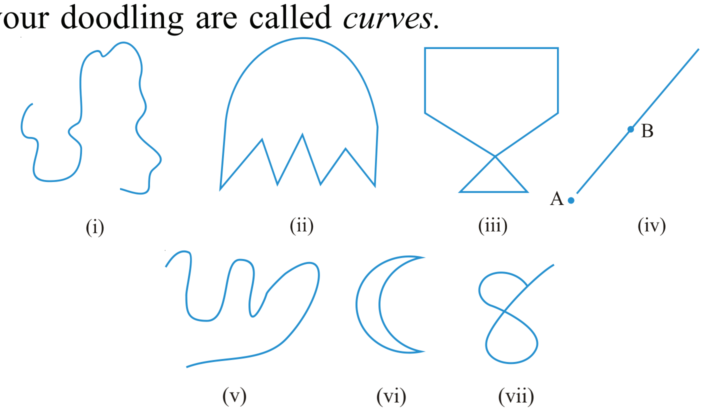


Fig 4.10

You can draw some of these drawings without lifting the pencil from the paper and without the use of a ruler. These are all curves (Fig 4.10).

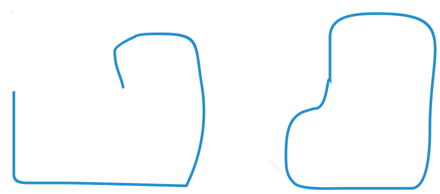
'Curve' in everyday usage means "not straight". In Mathematics, a curve can be straight like the one shown in fig 4.10 (iv).

Observe that the curves (iii) and (vii) in Fig 4.10 cross themselves, whereas the curves (i), (ii), (v) and (vi) in Fig 4.10 do not. If a curve does not cross itself, then it is called a **simple curve**.

Draw five more simple curves and five curves that are not simple.

Consider these now (Fig 4.11).

What is the difference between these two? The first i.e. Fig 4.11 (i) is an **open curve** and the second i.e. Fig 4.11(ii) is a **closed curve**. Can you identify some closed and open curves from the figures Fig 4.10 (i), (ii), (v), (vi)? Draw five curves each that are open and closed.



(i) (ii)

Fig 4.11

Position in a figure

A court line in a tennis court divides it into three parts : inside the line, on the line and outside the line. You cannot enter inside without crossing the line.

A compound wall separates your house from the road. You talk about 'inside' the compound, 'on' the boundary of the compound and 'outside' the compound.

In a closed curve, thus, there are three parts.

(i) interior ('inside') of the curve

(ii) boundary ('on') of the curve and

(iii) exterior ('outside') of the curve.

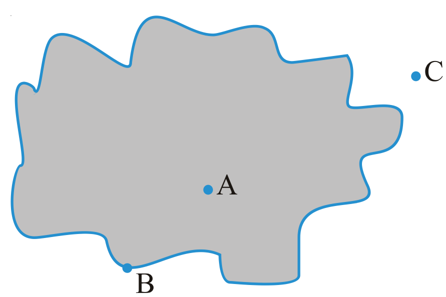


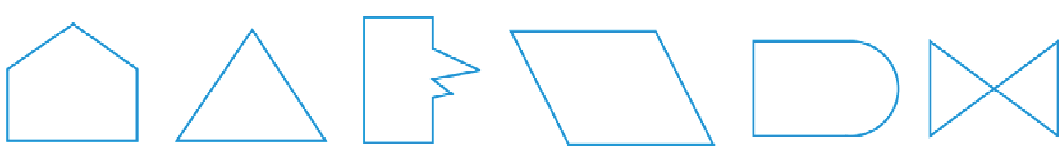
Fig 4.12

In the figure 4.12, A is in the interior, C is in the exterior and B is on the curve.

The interior of a curve together with its boundary is called its "**region**".

4.9 Polygons

Look at these figures 4.13 (i), (ii), (iii), (iv), (v) and (vi).



(i) (ii) (iii) (iv) (v) (vi)

Fig 4.13

What can you say? Are they closed? How does each one of them differ from the other? (i), (ii), (iii), (iv) and (vi) are special because they are made up entirely of line segments. Out of these (i), (ii), (iii) and (iv) are also simple closed curves. They are called **polygons.**

So, a figure is a polygon if it is a simple closed figure made up entirely of line segments. Draw ten differently shaped polygons.

Do This

Try to form a polygon with

1. Five matchsticks.
2. Four matchsticks.
3. Three matchsticks.
4. Two matchsticks.

In which case was it not possible? Why?

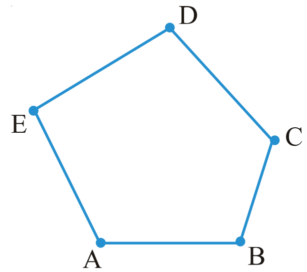


Fig 4.14

Sides, vertices and diagonals

Examine the figure given here (Fig 4.14).

Give justification to call it a polygon.

The line segments forming a polygon are called its *sides*.

What are the sides of polygon ABCDE? (Note how the corners are named in order.)

Sides are $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$ and $\overline{EA}$.

The meeting point of a pair of sides is called its *vertex*.

Sides $\overline{AE}$ and $\overline{ED}$ meet at E, so E is a vertex of the polygon ABCDE. Points B and C are its other vertices. Can you name the sides that meet at these points?

Can you name the other vertices of the above polygon ABCDE?

Any two sides with a common end point are called the *adjacent sides* of the polygon.

Are the sides $\overline{AB}$ and $\overline{BC}$ adjacent? How about $\overline{AE}$ and $\overline{DC}$?

The end points of the same side of a polygon are called the *adjacent vertices*. Vertices E and D are adjacent, whereas vertices A and D are not adjacent vertices. Do you see why?

Consider the pairs of vertices which are not adjacent. The joins of these vertices are called the diagonals of the polygon.

In the figure 4.15, $\overline{AC}$, $\overline{AD}$, $\overline{BD}$, $\overline{BE}$ and $\overline{CE}$ are diagonals.

Is $\overline{BC}$ a diagonal, Why or why not?

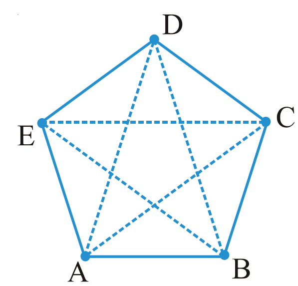


Fig 4.15

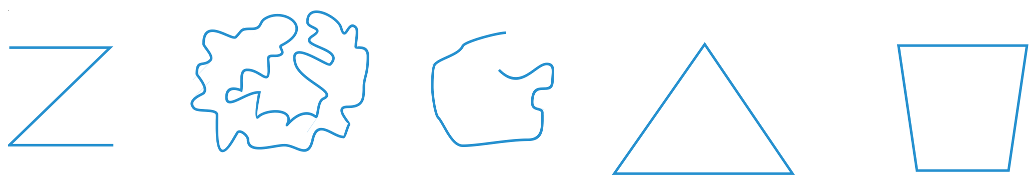
If you try to join adjacent vertices, will the result be a diagonal?

Name all the sides, adjacent sides, adjacent vertices of the figure ABCDE (Fig 4.15).

Draw a polygon ABCDEFGH and name all the sides, adjacent sides and vertices as well as the diagonals of the polygon.

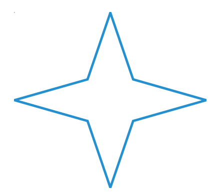
EXERCISE 4.2

1. Classify the following curves as (i) Open or (ii) Closed.



(a) (b) (c) (d) (e)

2. Draw rough diagrams to illustrate the following :
   (a) Open curve (b) Closed curve.
3. Draw any polygon and shade its interior.
4. Consider the given figure and answer the questions :
   (a) Is it a curve? (b) Is it closed?



5. Illustrate, if possible, each one of the following with a rough diagram:
   (a) A closed curve that is not a polygon.
   (b) An open curve made up entirely of line segments.
   (c) A polygon with two sides.

4.10 Angles

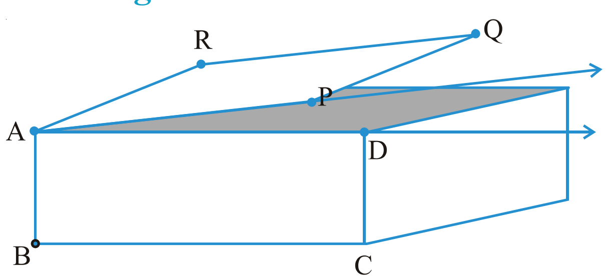


Fig 4.16

Angles are made when corners are formed.

Here is a picture (Fig 4.16) where the top of a box is like a hinged lid. The edges AD of the box and AP of the door can be imagined as two rays $\overrightarrow{AD}$ and $\overrightarrow{AP}$. These two rays have a common initial point A. The two rays here together are said to form an angle.

An angle is made up of two rays starting from a common initial point.

The two rays forming the angle are called the *arms* or *sides* of the angle.

The common initial point is the *vertex* of the angle.

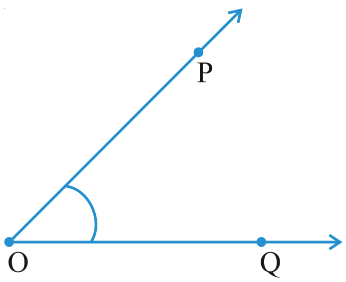


Fig 4.17

This is an angle formed by rays $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ (Fig 4.17). To show this we use a small curve at the vertex. (see Fig 4.17). O is the vertex. What are the sides? Are they not $\overrightarrow{OP}$ and $\overrightarrow{OQ}$? How can we name this angle? We can simply say that it is an angle at O. To be more specific we identify some two points, one on each side and the vertex to name the angle. Angle POQ is thus a better way of naming the angle. We denote this by $\angle POQ$.

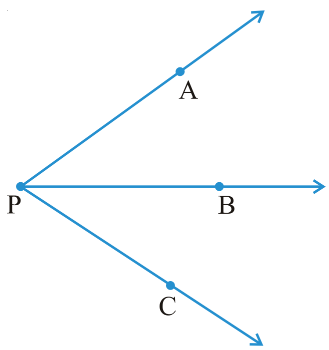


Fig 4.18

Think, discuss and write

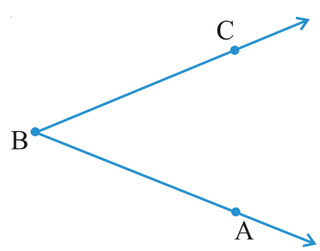
Look at the diagram (Fig 4.18). What is the name of the angle? Shall we say $\angle P$? But then which one do we mean? By $\angle P$ what do we mean? Is naming an angle by vertex helpful here? Why not?

By $\angle P$ we may mean $\angle APB$ or $\angle CPB$ or even $\angle APC$! We need more information.

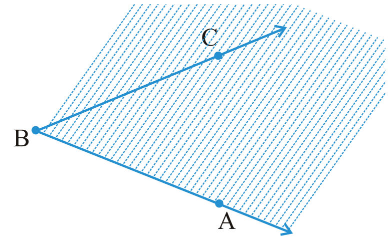
Note that in specifying the angle, the vertex is always written as the middle letter.

Do This

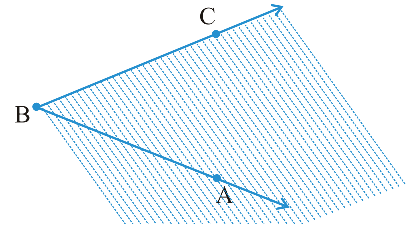
Take any angle, say $\angle ABC$.



Shade that portion of the paper bordering $\overrightarrow{BA}$ and where $\overrightarrow{BC}$ lies.



Now shade in a different colour the portion of the paper bordering $\overrightarrow{BC}$ and where $\overrightarrow{BA}$ lies.



The portion common to both shadings is called the interior of $\angle ABC$ (Fig 4.19). (Note that **the interior** is not a restricted area; it extends indefinitely since the two sides extend indefinitely).

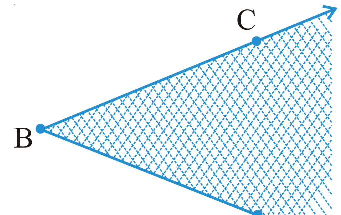


Fig 4.19

In this diagram (Fig 4.20), X is in the interior of the angle, Z is not in the interior but in the exterior of the angle; and S is on the $\angle PQR$. Thus, the angle also has three parts associated with it.

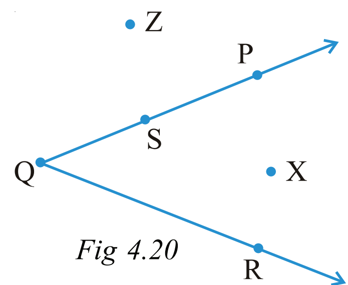
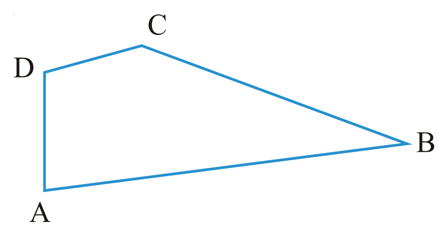


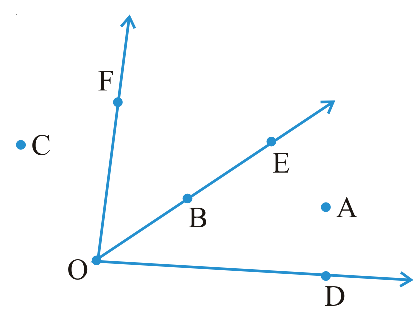
Fig 4.20

EXERCISE 4.3

1. Name the angles in the given figure.



2. In the given diagram, name the point(s)
   (a) In the interior of $\angle DOE$
   (b) In the exterior of $\angle EOF$
   (c) On $\angle EOF$



3. Draw rough diagrams of two angles such that they have
   (a) One point in common.
   (b) Two points in common.
   (c) Three points in common.
   (d) Four points in common.
   (e) One ray in common.

4.11 Triangles

A triangle is a three-sided polygon. In fact, it is the polygon with the least number of sides.

Look at the triangle in the diagram (Fig 4.21). We write ΔABC instead of writing "Triangle ABC".

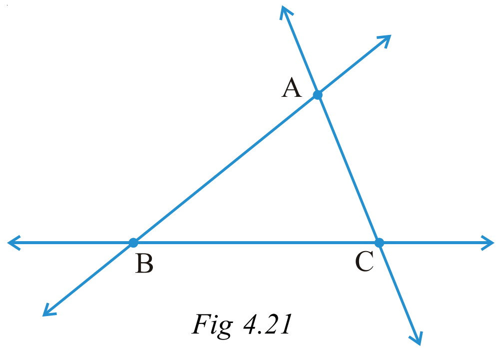


Fig 4.21

In ΔABC, how many sides and how many angles are there?

The three sides of the triangle are $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$. The three angles are $\angle BAC$, $\angle BCA$ and $\angle ABC$. The points A, B and C are called the vertices of the triangle.

Being a polygon, a triangle has an exterior and an interior. In the figure 4.22, P is in the interior of the triangle, R is in the exterior and Q on the triangle.

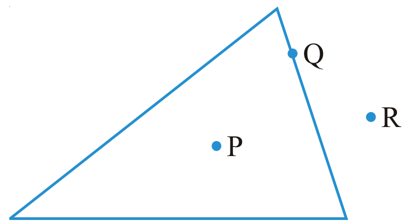
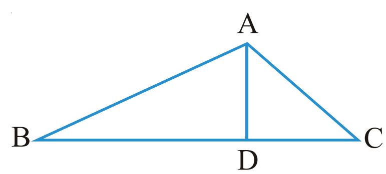


Fig 4.22

EXERCISE 4.4

1. Draw a rough sketch of a triangle ABC. Mark a point P in its interior and a point Q in its exterior. Is the point A in its exterior or in its interior?
2. (a) Identify three triangles in the figure.
   (b) Write the names of seven angles.
   (c) Write the names of six line segments.
   (d) Which two triangles have $\angle B$ as common?



4.12 Quadrilaterals

A four sided polygon is a *quadrilateral*. It has 4 sides and 4 angles. As in the case of a triangle, you can visualise its interior too.

Note the cyclic manner in which the vertices are named.

This quadrilateral ABCD (Fig 4.23) has four sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$. It has four angles $\angle A$, $\angle B$, $\angle C$ and $\angle D$.

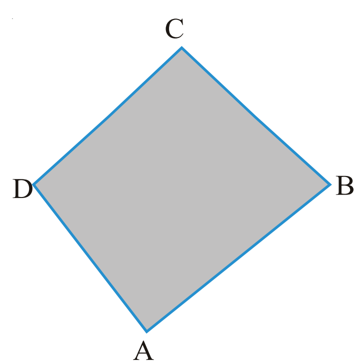
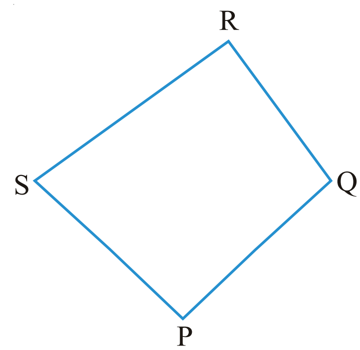
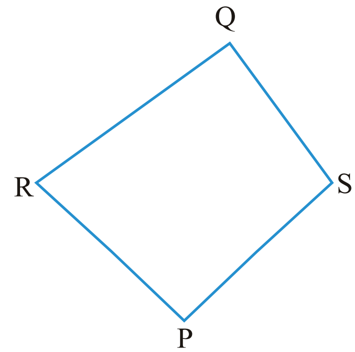


Fig 4.23

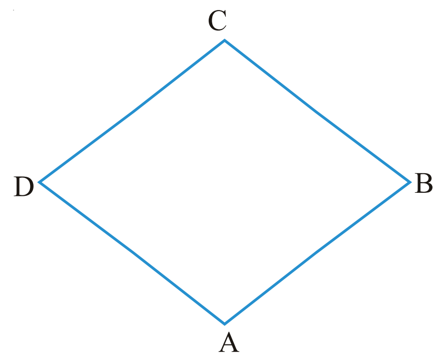


This is quadrilateral PQRS.



Is this quadrilateral PQRS?

In any quadrilateral ABCD, $\overline{AB}$ and $\overline{BC}$ are *adjacent sides*. Can you write other pairs of adjacent sides?

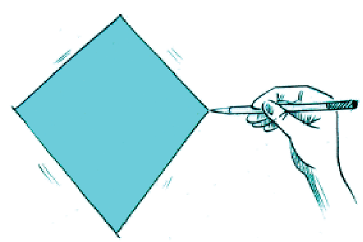


$\overline{AB}$ and $\overline{DC}$ are *opposite sides*; Name the other pair of opposite sides.

$\angle A$ and $\angle C$ are said to be *opposite angles*; similarly, $\angle D$ and $\angle B$ are opposite angles. Naturally $\angle A$ and $\angle B$ are *adjacent angles*. You can now list other pairs of adjacent angles.

EXERCISE 4.5

1. Draw a rough sketch of a quadrilateral PQRS. Draw its diagonals. Name them. Is the meeting point of the diagonals in the interior or exterior of the quadrilateral?



2. Draw a rough sketch of a quadrilateral KLMN. State,
   (a) two pairs of opposite sides,
   (b) two pairs of opposite angles,
   (c) two pairs of adjacent sides,
   (d) two pairs of adjacent angles.
3. **Investigate :**
   Use strips and fasteners to make a triangle and a quadrilateral.
   Try to push inward at any one vertex of the triangle. Do the same to the quadrilateral.
   Is the triangle distorted? Is the quadrilateral distorted? Is the triangle rigid?
   Why is it that structures like electric towers make use of triangular shapes and not quadrilaterals?

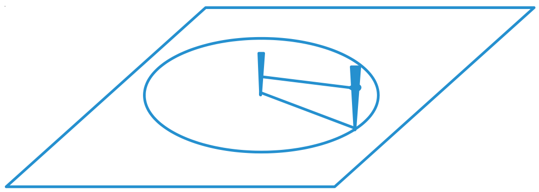
4.13 Circles

In our environment, you find many things that are round, a wheel, a bangle, a coin etc. We use the round shape in many ways. It is easier to roll a heavy steel tube than to drag it.

A circle is a simple closed curve which is not a polygon. It has some very special properties.

Do This

- Place a bangle or any round shape; trace around to get a circular shape.
- If you want to make a circular garden, how will you proceed?



Take two sticks and a piece of rope. Drive one stick into the ground. This is the centre of the proposed circle. Form two loops, one at each end of the rope. Place one loop around the stick at the centre. Put the other around the other stick. Keep the sticks vertical to the ground. Keep the rope taut all the time and trace the path. You get a circle.

Naturally every point on the circle is at equal distance from the centre.

Parts of a circle

Here is a circle with *centre* C (Fig 4.24)

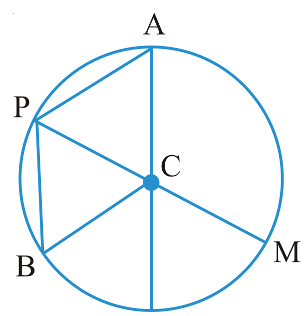


Fig 4.24

A, P, B, M are points on the circle. You will see that

CA = CP = CB = CM.

Each of the segments $\overline{CA}$, $\overline{CP}$, $\overline{CB}$, $\overline{CM}$ is *radius* of the circle. The radius is a line segment that connects the centre to a point on the circle. $\overline{CP}$ and $\overline{CM}$ are radii (plural of 'radius') such that C, P, M are in a line. $\overline{PM}$ is known as *diameter* of the circle.

Is a diameter double the size of a radius? Yes.

$\overline{PB}$ is a *chord* connecting two points on a circle. Is $\overline{PM}$ also a chord?

An arc is a portion of circle.

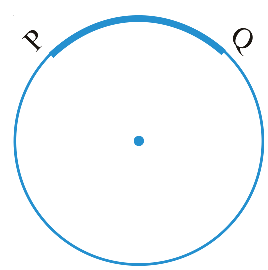


Fig 4.25

If P and Q are two points you get the arc PQ. We write it as $\overparen{PQ}$ (Fig 4.25).

As in the case of any simple closed curve you can think of the *interior* and *exterior* of a circle. A region in the interior of a circle enclosed by an arc on one side and a pair of radii on the other two sides is called a *sector* (Fig 4.26).

A region in the interior of a circle enclosed by a chord and an arc is called a *segment* of the circle.

Take any circular object. Use a thread and wrap it around the object once. The length of the thread is the distance covered to travel around the object once. What does this length denote?

The distance around a circle is its *circumference*.

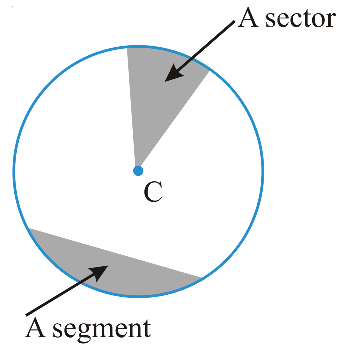
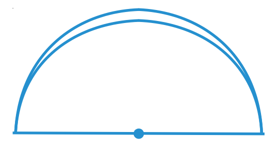
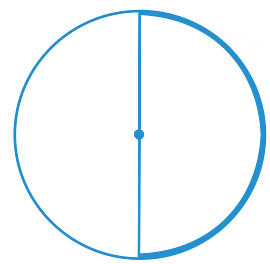


Fig 4.26

Do This

- Take a circular sheet. Fold it into two halves. Crease the fold and open up. Do you find that the circular region is halved by the diameter?

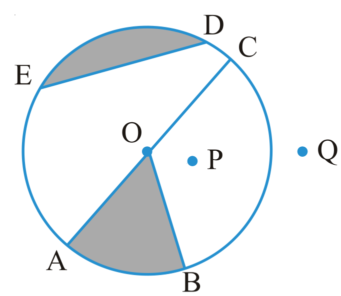




A diameter of a circle divides it into two equal parts; each part is a *semi-circle*. A semi-circle is half of a circle, with the end points of diameter as part of the boundary.

EXERCISE 4.6

1. From the figure, identify :
   (a) the centre of circle (b) three radii
   (c) a diameter (d) a chord
   (e) two points in the interior (f) a point in the exterior
   (g) a sector (h) a segment



2. (a) Is every diameter of a circle also a chord?
   (b) Is every chord of a circle also a diameter?
3. Draw any circle and mark
   (a) its centre (b) a radius
   (c) a diameter (d) a sector
   (e) a segment (f) a point in its interior
   (g) a point in its exterior (h) an arc
4. Say true or false :
   (a) Two diameters of a circle will necessarily intersect.
   (b) The centre of a circle is always in its interior.

What have we discussed?

1. A point determines a location. It is usually denoted by a capital letter.
2. A line segment corresponds to the shortest distance between two points. The line segment joining points A and B is denoted by $\overline{AB}$.

3. A line is obtained when a line segment like $\overline{AB}$ is extended on both sides indefinitely; it is denoted by $\overleftrightarrow{AB}$ or sometimes by a single small letter like *l*.
4. Two distinct lines meeting at a point are called *intersecting lines*.
5. Two lines in a plane are said to be parallel if they do not meet.
6. A ray is a portion of line starting at a point and going in one direction endlessly.
7. Any drawing (straight or non-straight) done without lifting the pencil may be called a curve. In this sense, a line is also a curve.
8. A simple curve is one that does not cross itself.
9. A curve is said to be closed if its ends are joined; otherwise it is said to be open.
10. A polygon is a simple closed curve made up of line segments. Here,
    (i) The line segments are the sides of the polygon.
    (ii) Any two sides with a common end point are adjacent sides.
    (iii) The meeting point of a pair of sides is called a *vertex*.
    (iv) The end points of the same side are adjacent vertices.
    (v) The join of any two non-adjacent vertices is a diagonal.
11. An angle is made up of two rays starting from a common starting/initial point.

    Two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ make $\angle AOB$ (or also called $\angle BOA$).

    An angle leads to three divisions of a region:

    On the angle, the interior of the angle and the exterior of the angle.
12. A triangle is a three-sided polygon.
13. A quadrilateral is a four-sided polygon. (It should be named cyclically).

    In any quadrilateral ABCD, $\overline{AB}$ & $\overline{DC}$ and $\overline{AD}$ & $\overline{BC}$ are pairs of opposite sides. $\angle A$ & $\angle C$ and $\angle B$ & $\angle D$ are pairs of opposite angles. $\angle A$ is adjacent to $\angle B$ & $\angle D$; similar relations exist for other three angles.
14. A circle is the path of a point moving at the same distance from a fixed point.

    The fixed point is the centre, the fixed distance is the radius and the distance around the circle is the *circumference*.

    A *chord* of a circle is a line segment joining any two points on the circle.

    A *diameter* is a chord passing through the centre of the circle.

    A sector is the *region* in the interior of a circle enclosed by an arc on one side and a pair of radii on the other two sides.

    A *segment* of a circle is a region in the interior of the circle enclosed by an arc and a chord.

    The diameter of a circle divides it into *two semi-circles*.

Chapter 5

Understanding Elementary Shapes

5.1 Introduction

All the shapes we see around us are formed using curves or lines. We can see corners, edges, planes, open curves and closed curves in our surroundings. We organise them into line segments, angles, triangles, polygons and circles. We find that they have different sizes and measures. Let us now try to develop tools to compare their sizes.

5.2 Measuring Line Segments

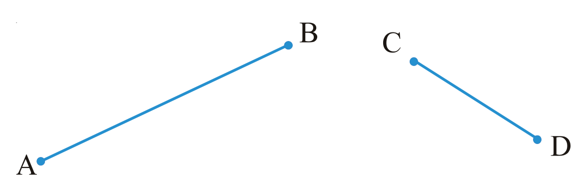
We have drawn and seen so many line segments. A triangle is made of three, a quadrilateral of four line segments.

A line segment is a fixed portion of a line. This makes it possible to measure a line segment. This measure of each line segment is a unique number called its "length". We use this idea to compare line segments.

To compare any two line segments, we find a relation between their lengths. This can be done in several ways.

(i) Comparison by observation:

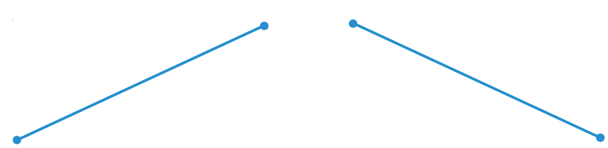
By just looking at them can you tell which one is longer?



You can see that $\overline{AB}$ is longer.

But you cannot always be sure about your usual judgment.

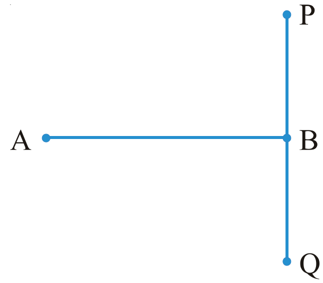
For example, look at the adjoining segments :



The difference in lengths between these two may not be obvious. This makes other ways of comparing necessary.

In this adjacent figure, $\overline{AB}$ and $\overline{PQ}$ have the same lengths. This is not quite obvious.

So, we need better methods of comparing line segments.



(ii) Comparison by Tracing

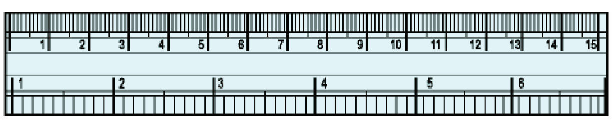
To compare $\overline{AB}$ and $\overline{CD}$, we use a tracing paper, trace $\overline{CD}$ and place the traced segment on $\overline{AB}$.

Can you decide now which one among $\overline{AB}$ and $\overline{CD}$ is longer?

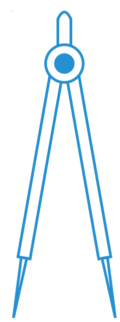
The method depends upon the accuracy in tracing the line segment. Moreover, if you want to compare with another length, you have to trace another line segment. This is difficult and you cannot trace the lengths everytime you want to compare them.

(iii) Comparison using Ruler and a Divider

Have you seen or can you recognise all the instruments in your instrument box? Among other things, you have a ruler and a divider.



Ruler

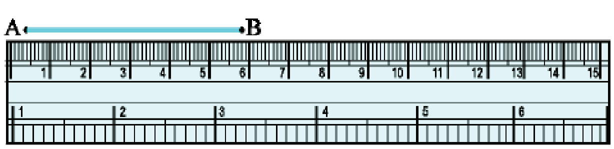


Divider

Note how the ruler is marked along one of its edges. It is divided into 15 parts. Each of these 15 parts is of length 1cm.

Each centimetre is divided into 10subparts. Each subpart of the division of a cm is 1mm.

1 mm is 0.1 cm.
2 mm is 0.2 cm and so on .
2.3 cm will mean 2 cm and 3 mm.



How many millimetres make one centimetre? Since 1cm = 10 mm, how will we write 2 cm? 3mm? What do we mean by 7.7 cm?

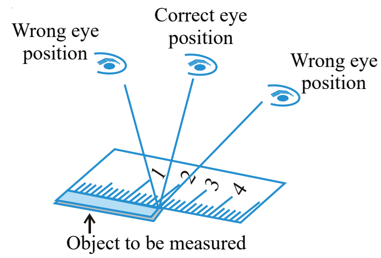
Place the zero mark of the ruler at A. Read the mark against B. This gives the length of $\overline{AB}$. Suppose the length is 5.8 cm, we may write,

Length AB = 5.8 cm or more simply as AB = 5.8 cm.

There is room for errors even in this procedure. The thickness of the ruler may cause difficulties in reading off the marks on it.

Think, discuss and write

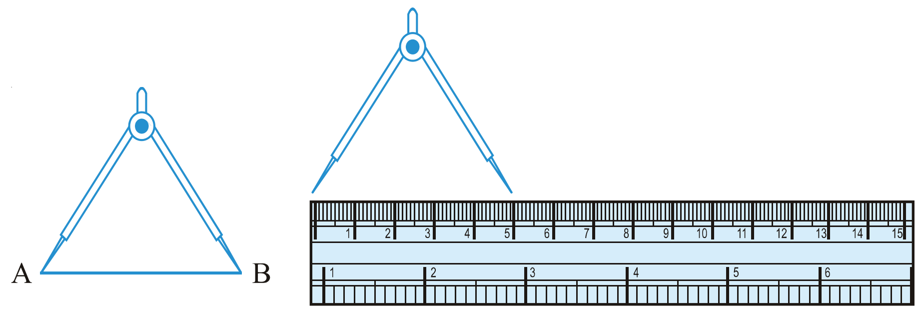
1. What other errors and difficulties might we face?
2. What kind of errors can occur if viewing the mark on the ruler is not proper? How can one avoid it?



Positioning error

To get correct measure, the eye should be correctly positioned, just vertically above the mark. Otherwise errors can happen due to angular viewing.

Can we avoid this problem? Is there a better way?
Let us use the divider to measure length.



Open the divider. Place the end point of one of its arms at A and the end point of the second arm at B. Taking care that opening of the divider is not disturbed, lift the divider and place it on the ruler. Ensure that one end point is at the zero mark of the ruler. Now read the mark against the other end point.

Try These

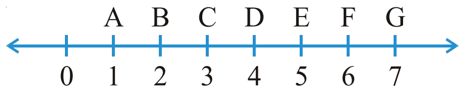
1. Take any post card. Use the above technique to measure its two adjacent sides.
2. Select any three objects having a flat top. Measure all sides of the top using a divider and a ruler.

EXERCISE 5.1

1. What is the disadvantage in comparing line segments by mere observation?
2. Why is it better to use a divider than a ruler, while measuring the length of a line segment?
3. Draw any line segment, say $\overline{AB}$. Take any point C lying in between A and B. Measure the lengths of AB, BC and AC. Is AB = AC + CB?

   [Note : If A,B,C are any three points on a line such that AC + CB = AB, then we can be sure that C lies between A and B.]

4. If A,B,C are three points on a line such that AB = 5 cm, BC = 3 cm and AC = 8 cm, which one of them lies between the other two?
5. Verify, whether D is the mid point of $\overline{AG}$.



6. If B is the mid point of $\overline{AC}$ and C is the mid point of $\overline{BD}$, where A,B,C,D lie on a straight line, say why AB = CD?
7. Draw five triangles and measure their sides. Check in each case, if the sum of the lengths of any two sides is always less than the third side.

5.3 Angles – 'Right' and 'Straight'

You have heard of directions in Geography. We know that China is to the north of India, Sri Lanka is to the south. We also know that Sun rises in the east and sets in the west. There are four main directions. They are North (N), South (S), East (E) and West (W).

Do you know which direction is opposite to north?

Which direction is opposite to west?

Just recollect what you know already. We now use this knowledge to learn a few properties about angles.

Stand facing north.

Do This

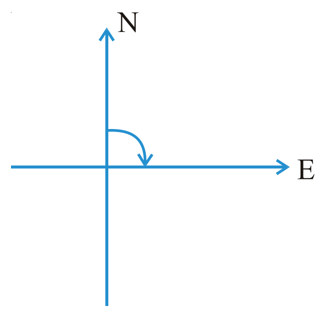
Turn clockwise to east.

We say, you have turned through a **right angle.**

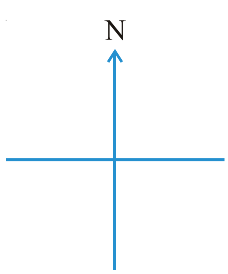
Follow this by a 'right-angle-turn', clockwise.

You now face south.

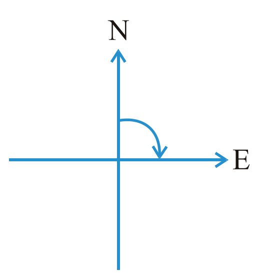
If you turn by a right angle in the anti-clockwise direction, which direction will you face? It is east again! (Why?)



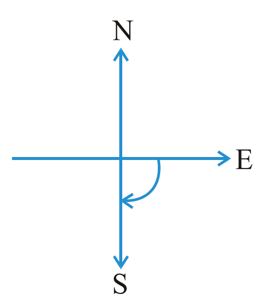
Study the following positions :



You stand facing north



By a 'right-angle-turn' clockwise, you now face east

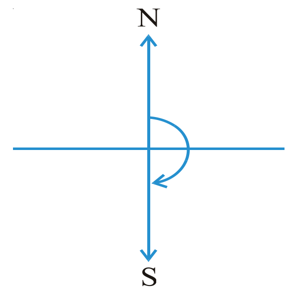


By another 'right-angle-turn' you finally face south.

From facing north to facing south, you have turned by two right angles. Is not this the same as a single turn by two right angles?

The turn from north to east is by a right angle.

The turn from north to south is by two right angles; it is called a **straight angle**. (NS is a straight line!)

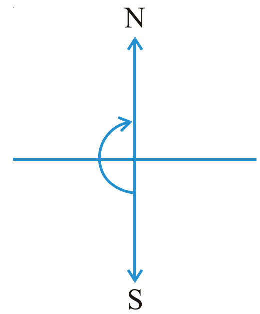


Stand facing south.

Turn by a straight angle.

Which direction do you face now?

You face north!

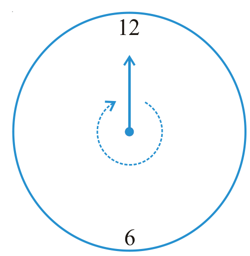


To turn from north to south, you took a straight angle turn, again to turn from south to north, you took another straight angle turn in the same direction. Thus, turning by two straight angles you reach your original position.

Think, discuss and write

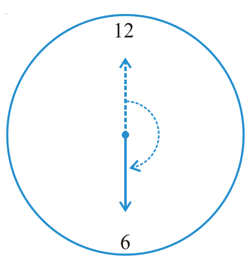
By how many right angles should you turn in the same direction to reach your original position?

Turning by two straight angles (or four right angles) in the same direction makes a full turn. This one complete turn is called one revolution. The angle for one revolution is a **complete angle**.



We can see such revolutions on clock-faces. When the hand of a clock moves from one position to another, it turns through an **angle.**

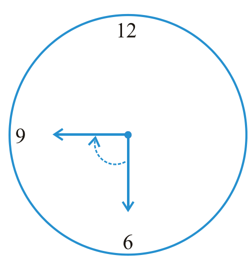
Suppose the hand of a clock starts at 12 and goes round until it reaches at 12 again. Has it not made one revolution? So, how many right angles has it moved? Consider these examples :



From 12 to 6

$\frac{1}{2}$ of a revolution.

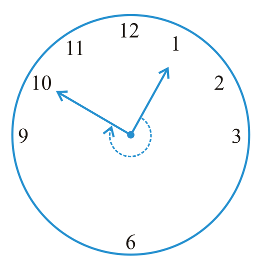
or 2 right angles.



From 6 to 9

$\frac{1}{4}$ of a revolution

or 1 right angle.



From 1 to 10

$\frac{3}{4}$ of a revolution

or 3 right angles.

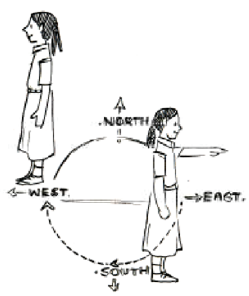
Try These

1. What is the angle name for half a revolution?
2. What is the angle name for one-fourth revolution?
3. Draw five other situations of one-fourth, half and three-fourth revolution on a clock.

Note that there is no special name for three-fourth of a revolution.

EXERCISE 5.2

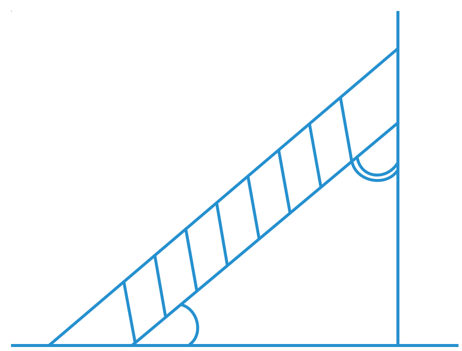
1. What fraction of a clockwise revolution does the hour hand of a clock turn through, when it goes from
   (a) 3 to 9 (b) 4 to 7 (c) 7 to 10
   (d) 12 to 9 (e) 1 to 10 (f) 6 to 3
2. Where will the hand of a clock stop if it
   (a) starts at 12 and makes $\frac{1}{2}$ of a revolution, clockwise?
   (b) starts at 2 and makes $\frac{1}{2}$ of a revolution, clockwise?
   (c) starts at 5 and makes $\frac{1}{4}$ of a revolution, clockwise?
   (d) starts at 5 and makes $\frac{3}{4}$ of a revolution, clockwise?
3. Which direction will you face if you start facing
   (a) east and make $\frac{1}{2}$ of a revolution clockwise?
   (b) east and make $1\frac{1}{2}$ of a revolution clockwise?
   (c) west and make $\frac{3}{4}$ of a revolution anti-clockwise?
   (d) south and make one full revolution?
   (Should we specify clockwise or anti-clockwise for this last question? Why not?)



4. What part of a revolution have you turned through if you stand facing
   (a) east and turn clockwise to face north?
   (b) south and turn clockwise to face east?
   (c) west and turn clockwise to face east?
5. Find the number of right angles turned through by the hour hand of a clock when it goes from
   (a) 3 to 6 (b) 2 to 8 (c) 5 to 11
   (d) 10 to 1 (e) 12 to 9 (f) 12 to 6

6. How many right angles do you make if you start facing
   (a) south and turn clockwise to west?
   (b) north and turn anti-clockwise to east?
   (c) west and turn to west?
   (d) south and turn to north?
7. Where will the hour hand of a clock stop if it starts
   (a) from 6 and turns through 1 right angle?
   (b) from 8 and turns through 2 right angles?
   (c) from 10 and turns through 3 right angles?
   (d) from 7 and turns through 2 straight angles?

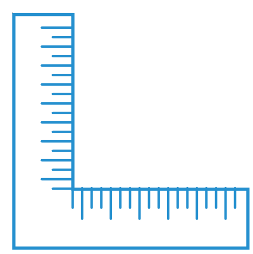
5.4 Angles – 'Acute', 'Obtuse' and 'Reflex'



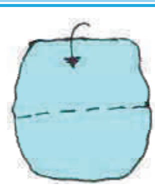
We saw what we mean by a right angle and a straight angle. However, not all the angles we come across are one of these two kinds. The angle made by a ladder with the wall (or with the floor) is neither a right angle nor a straight angle.

Think, discuss and write

Are there angles smaller than a right angle?
Are there angles greater than a right angle?
Have you seen a carpenter's square? It looks like the letter "L" of English alphabet. He uses it to check right angles. Let us also make a similar 'tester' for a right angle.



Do This



Step 1
Take a piece of paper

Step 2
Fold it somewhere in the middle

Step 3
Fold again the straight edge. Your tester is ready

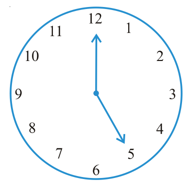
Observe your improvised 'right-angle-tester'. [Shall we call it RA tester?] Does one edge end up straight on the other?

Suppose any shape with corners is given. You can use your RA tester to test the angle at the corners.

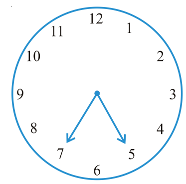
Do the edges match with the angles of a paper? If yes, it indicates a right angle.

Try These

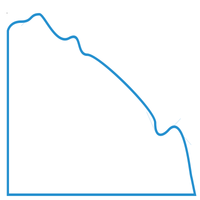
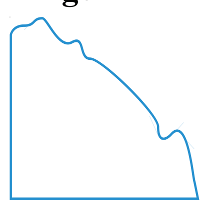
1. The hour hand of a clock moves from 12 to 5. Is the revolution of the hour hand more than 1 right angle?



2. What does the angle made by the hour hand of the clock look like when it moves from 5 to 7. Is the angle moved more than 1 right angle?

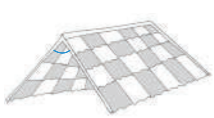


3. Draw the following and check the angle with your RA tester.
   (a) going from 12 to 2 (b) from 6 to 7
   (c) from 4 to 8 (d) from 2 to 5
4. Take five different shapes with corners. Name the corners. Examine them with your tester and tabulate your results for each case :

Corner	Smaller than 	Larger than 
A	....	....
B	....	....
C	....	....
⋮		

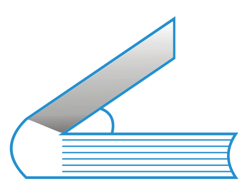
Other names

- An angle smaller than a right angle is called an **acute angle**. These are acute angles.



Roof top

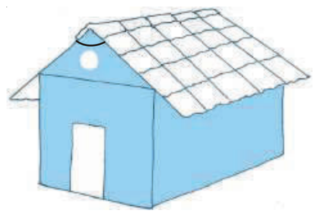
Sea-saw



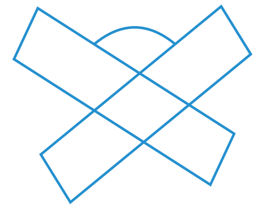
Opening book

Do you see that each one of them is less than one-fourth of a revolution? Examine them with your RA tester.

- If an angle is larger than a right angle, but less than a straight angle, it is called an **obtuse angle**. These are obtuse angles.



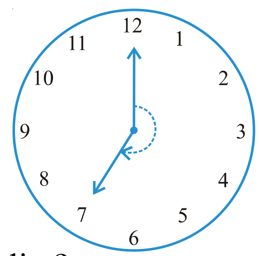
House



Book reading desk

Do you see that each one of them is greater than one-fourth of a revolution but less than half a revolution? Your RA tester may help to examine.

Identify the obtuse angles in the previous examples too.



- A reflex angle is larger than a straight angle.

  It looks like this. (See the angle mark)

  Were there any reflex angles in the shapes you made earlier?

  How would you check for them?

Try These

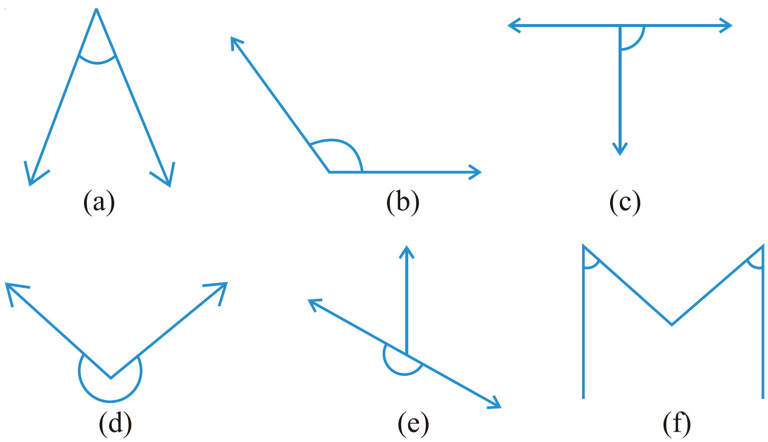
1. Look around you and identify edges meeting at corners to produce angles. List ten such situations.
2. List ten situations where the angles made are acute.
3. List ten situations where the angles made are right angles.
4. Find five situations where obtuse angles are made.
5. List five other situations where reflex angles may be seen.

EXERCISE 5.3

1. Match the following :

(i) Straight angle	(a) Less than one-fourth of a revolution
(ii) Right angle	(b) More than half a revolution
(iii) Acute angle	(c) Half of a revolution
(iv) Obtuse angle	(d) One-fourth of a revolution
(v) Reflex angle	(e) Between $\frac{1}{4}$ and $\frac{1}{2}$ of a revolution
	(f) One complete revolution

2. Classify each one of the following angles as right, straight, acute, obtuse or reflex :



(a) (b) (c) (d) (e) (f)

5.5 Measuring Angles

The improvised 'Right-angle tester' we made is helpful to compare angles with a right angle. We were able to classify the angles as acute, obtuse or reflex.

But this does not give a precise comparison. It cannot find which one among the two obtuse angles is greater. So in order to be more precise in comparison, we need to 'measure' the angles. We can do it with a 'protractor'.

The measure of angle

We call our measure, 'degree measure'. One complete revolution is divided into 360 equal parts. Each part is a **degree**. We write 360° to say 'three hundred sixty degrees'.

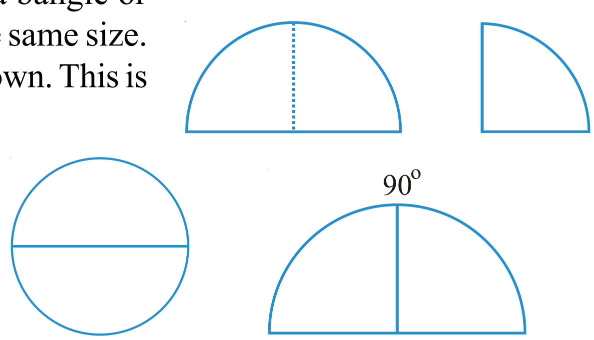
Think, discuss and write

How many degrees are there in half a revolution? In one right angle? In one straight angle?

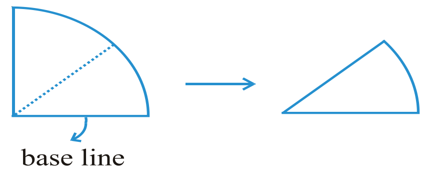
How many right angles make 180°? 360°?

Do This

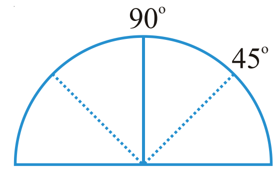
1. Cut out a circular shape using a bangle or take a circular sheet of about the same size.
2. Fold it twice to get a shape as shown. This is called a quadrant.
3. Open it out. You will find a semi-circle with a fold in the middle. Mark 90° on the fold.
4. Fold the semicircle to reach



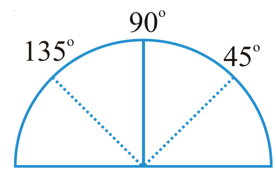
the quadrant. Now fold the quadrant once more as shown. The angle is half of 90° i.e. 45°.



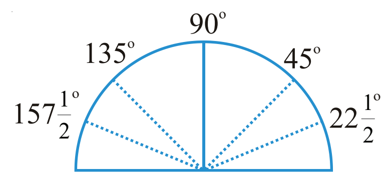
5. Open it out now. Two folds appear on each side. What is the angle upto the first new line? Write 45° on the first fold to the left of the base line.



6. The fold on the other side would be 90° + 45° = 135°



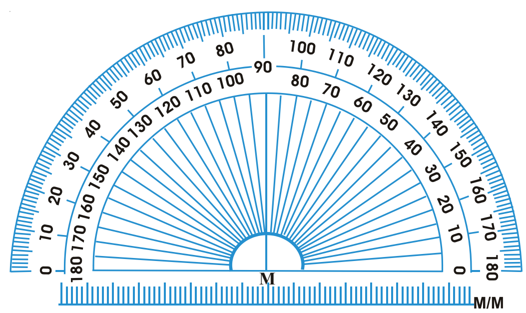
7. Fold the paper again upto 45° (half of the quadrant). Now make half of this. The first fold to the left of the base line now is half of 45° i.e. $22\frac{1}{2}^{\circ}$. The angle on the left of 135° would be $157\frac{1}{2}^{\circ}$.



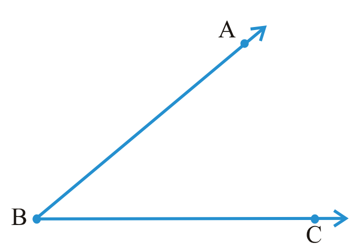
You have got a ready device to measure angles. This is an approximate protractor.

The Protractor

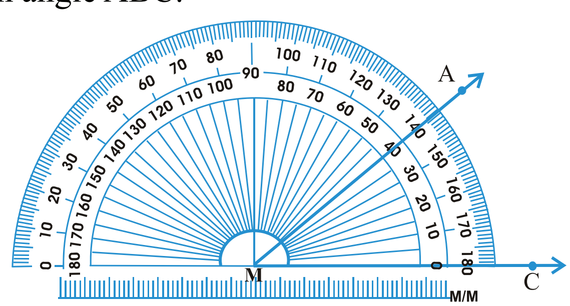
You can find a readymade protractor in your 'instrument box'. The curved edge is divided into 180 equal parts. Each part is equal to a 'degree'. The markings start from 0° on the right side and ends with 180° on the left side, and vice-versa.



Suppose you want to measure an angle ABC.



Given ∠ABC



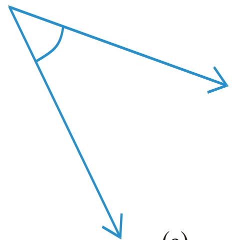
Measuring ∠ABC

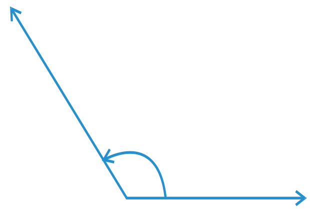
1. Place the protractor so that the mid point (M in the figure) of its straight edge lies on the vertex B of the angle.
2. Adjust the protractor so that $\overline{BC}$ is along the straight-edge of the protractor.
3. There are two 'scales' on the protractor : read that scale which has the 0° mark coinciding with the straight-edge (i.e. with ray $\overrightarrow{BC}$).
4. The mark shown by $\overline{BA}$ on the curved edge gives the degree measure of the angle.

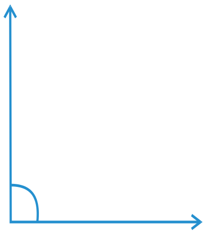
   We write m $\angle ABC = 40°$, or simply $\angle ABC = 40°$.

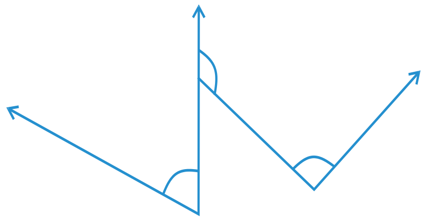
EXERCISE 5.4

1. What is the measure of (i) a right angle? (ii) a straight angle?
2. Say True or False :
   (a) The measure of an acute angle < 90°.
   (b) The measure of an obtuse angle < 90°.
   (c) The measure of a reflex angle > 180°.
   (d) The measure of one complete revolution = 360°.
   (e) If $m\angle A = 53°$ and $m\angle B = 35°$, then $m\angle A > m\angle B$.
3. Write down the measures of
   (a) some acute angles. (b) some obtuse angles.
   (give at least two examples of each).
4. Measure the angles given below using the Protractor and write down the measure.


(a)


(b)

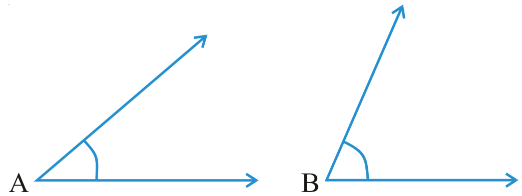

(c)


(d)

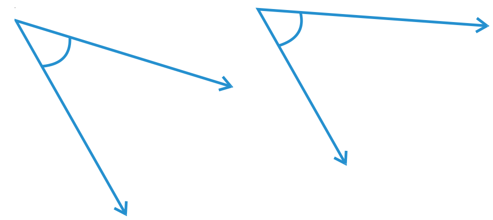
5. Which angle has a large measure?
First estimate and then measure.

Measure of Angle A =

Measure of Angle B =



6. From these two angles which has larger measure? Estimate and then confirm by measuring them.



7. Fill in the blanks with acute, obtuse, right or straight :

(a) An angle whose measure is less than that of a right angle is______.

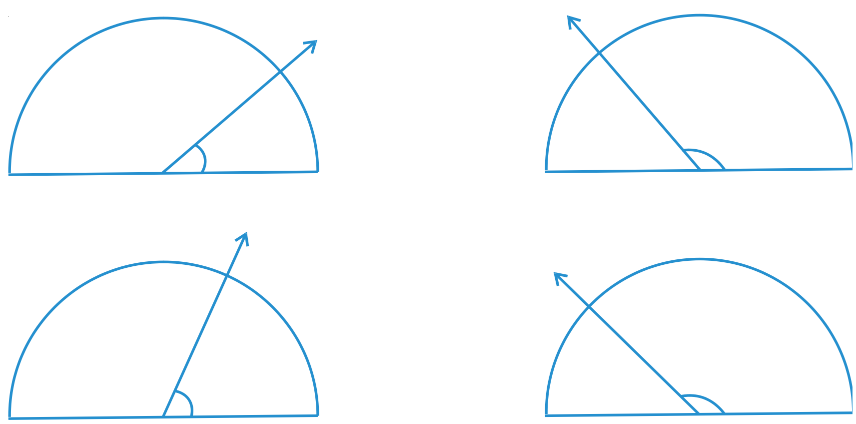
(b) An angle whose measure is greater than that of a right angle is ______.

(c) An angle whose measure is the sum of the measures of two right angles is _____.

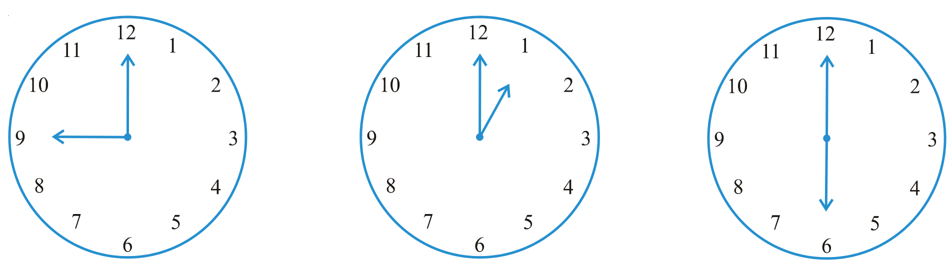
(d) When the sum of the measures of two angles is that of a right angle, then each one of them is ______.

(e) When the sum of the measures of two angles is that of a straight angle and if one of them is acute then the other should be _______.

8. Find the measure of the angle shown in each figure. (First estimate with your eyes and then find the actual measure with a protractor).



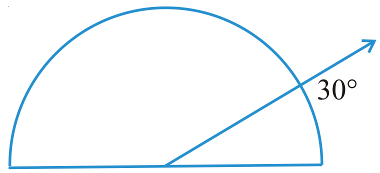
9. Find the angle measure between the hands of the clock in each figure :



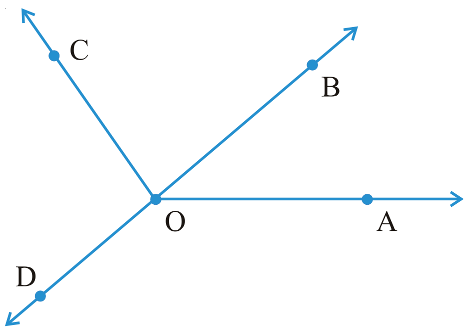
9.00 a.m. 1.00 p.m. 6.00 p.m.

10. **Investigate**

In the given figure, the angle measures 30°. Look at the same figure through a magnifying glass. Does the angle becomes larger? Does the size of the angle change?



11. Measure and classify each angle :



Angle	Measure	Type
$\angle AOB$		
$\angle AOC$		
$\angle BOC$		
$\angle DOC$		
$\angle DOA$		
$\angle DOB$		

5.6 Perpendicular Lines

When two lines intersect and the angle between them is a right angle, then the lines are said to be **perpendicular**. If a line AB is perpendicular to CD, we write $AB \perp CD$.

Think, discuss and write

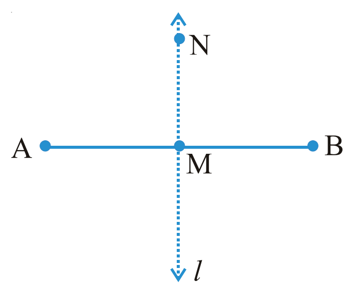
If $AB \perp CD$, then should we say that $CD \perp AB$ also?

Perpendiculars around us!

You can give plenty of examples from things around you for perpendicular lines (or line segments). The English alphabet T is one. Is there any other alphabet which illustrates perpendicularity?

Consider the edges of a post card. Are the edges perpendicular?

Let $\overline{AB}$ be a line segment. Mark its mid point as M. Let MN be a line perpendicular to $\overline{AB}$ through M.



Does MN divide $\overline{AB}$ into two equal parts?

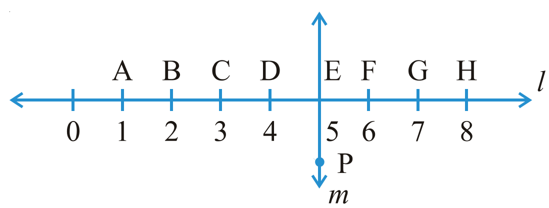
MN bisects $\overline{AB}$ (that is, divides $\overline{AB}$ into two equal parts) and is also perpendicular to $\overline{AB}$.

So we say MN is the **perpendicular bisector** of $\overline{AB}$.

You will learn to construct it later.

EXERCISE 5.5

1. Which of the following are models for perpendicular lines :
   (a) The adjacent edges of a table top.
   (b) The lines of a railway track.
   (c) The line segments forming the letter 'L'.
   (d) The letter V.
2. Let $\overline{PQ}$ be the perpendicular to the line segment $\overline{XY}$. Let $\overline{PQ}$ and $\overline{XY}$ intersect in the point A. What is the measure of $\angle PAY$?
3. There are two set-squares in your box. What are the measures of the angles that are formed at their corners? Do they have any angle measure that is common?
4. Study the diagram. The line l is perpendicular to line m
   (a) Is CE = EG?



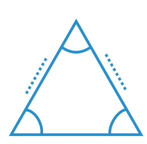
   (b) Does PE bisect CG?
   (c) Identify any two line segments for which PE is the perpendicular bisector.
   (d) Are these true?
      (i) AC > FG
      (ii) CD = GH
      (iii) BC < EH.

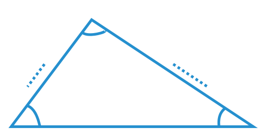
5.7 Classification of Triangles

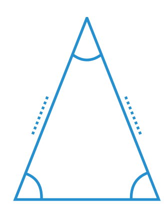
Do you remember a polygon with the least number of sides? That is a triangle. Let us see the different types of triangle we can get.

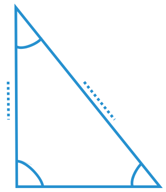
Do This

Using a protractor and a ruler find the measures of the sides and angles of the given triangles. Fill the measures in the given table.

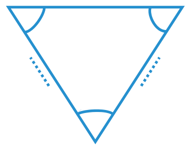

(a)


(b)

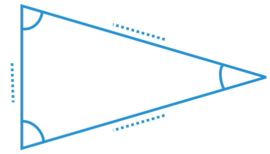

(c)



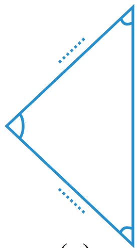
(d)



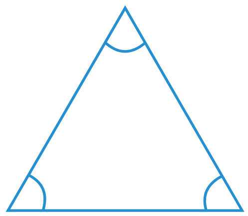
(e)



(f)



(g)



(h)

The measure of the angles of the triangle	What can you say about the angles?	Measures of the sides
(a) ...60^0..., ...60^0.., ...60^0.....,	All angles are equal	
(b) ...., ..., ...,	.... angles ....,	
(c) ...., ..., ...,	.... angles ....,	
(d) ...., ..., ...,	.... angles ....,	
(e) ...., ..., ...,	.... angles ....,	
(f) ...., ..., ...,	.... angles ....,	
(g) ...., ..., ...,	.... angles ....,	
(h) ...., ..., ...,	.... angles ....,	

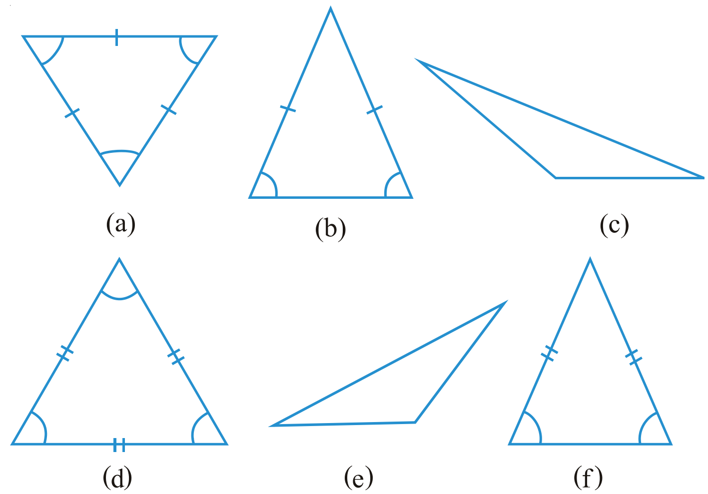
Observe the angles and the triangles as well as the measures of the sides carefully. Is there anything special about them?

What do you find?

- Triangles in which all the angles are equal.
  If all the angles in a triangle are equal, then its sides are also ....
- Triangles in which all the three sides are equal.
  If all the sides in a triangle are equal, then its angles are............. .
- Triangle which have two equal angles and two equal sides.
  If two sides of a triangle are equal, it has .... equal angles.
  and if two angles of a triangle are equal, it has .... equal sides.
- Triangles in which no two sides are equal.
  If none of the angles of a triangle are equal then none of the sides are equal.
  If the three sides of a triangle are unequal then, the three angles are also............. .

Take some more triangles and verify these. For this we will again have to measure all the sides and angles of the triangles.

The triangles have been divided into categories and given special names. Let us see what they are.



Naming triangles based on sides

A triangle having all three unequal sides is called a **Scalene Triangle** [(c), (e)].

A triangle having two equal sides is called an **Isosceles Triangle** [(b), (f)].

A triangle having three equal sides is called an **Equilateral Triangle** [(a), (d)].

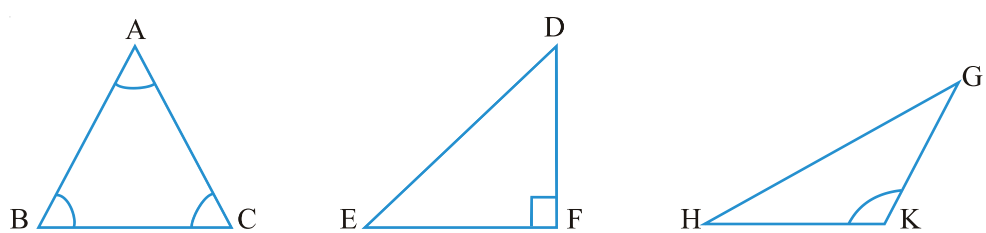
Classify all the triangles whose sides you measured earlier, using these definitions.

Naming triangles based on angles

If each angle is less than 90°, then the triangle is called an **acute angled triangle**.

If any one angle is a right angle then the triangle is called a **right angled triangle**.

If any one angle is greater than 90°, then the triangle is called an **obtuse angled triangle**.



Acute Angled Triangle   Right Angled Triangle   Obtuse Angled Triangle

Name the triangles whose angles were measured earlier according to these three categories. How many were right angled triangles?

Do This

Try to draw rough sketches of

(a) a scalene acute angled triangle.

(b) an obtuse angled isosceles triangle.

(c) a right angled isosceles triangle.

(d) a scalene right angled triangle.

Do you think it is possible to sketch

(a) an obtuse angled equilateral triangle ?

(b) a right angled equilateral triangle ?

(c) a triangle with two right angles?

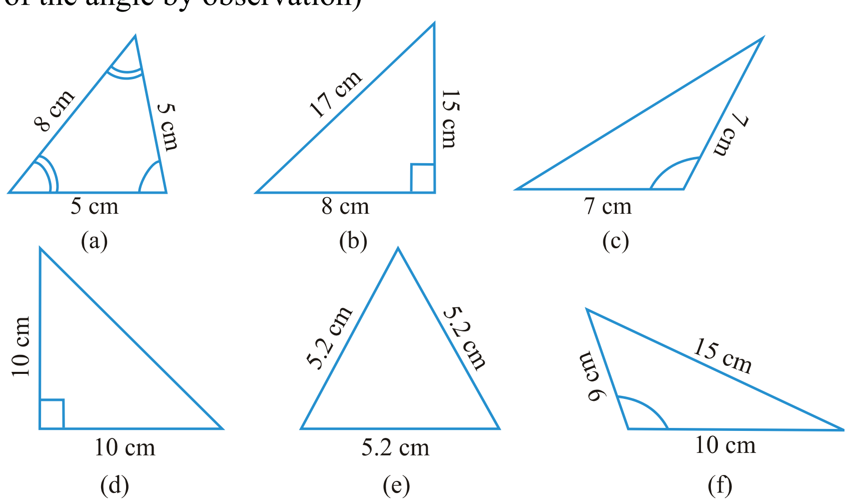
Think, discuss and write your conclusions.

EXERCISE 5.6

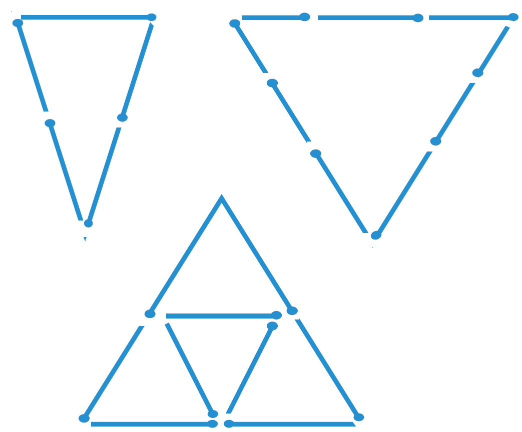
1. Name the types of following triangles :
   (a) Triangle with lengths of sides 7 cm, 8 cm and 9 cm.
   (b) ΔABC with AB = 8.7 cm, AC = 7 cm and BC = 6 cm.
   (c) ΔPQR such that PQ = QR = PR = 5 cm.
   (d) ΔDEF with $m\angle D = 90°$
   (e) ΔXYZ with $m\angle Y = 90°$ and XY = YZ.
   (f) ΔLMN with $m\angle L = 30°$, $m\angle M = 70°$ and $m\angle N = 80°$.
2. Match the following :

Measures of Triangle	Type of Triangle
(i) 3 sides of equal length	(a) Scalene
(ii) 2 sides of equal length	(b) Isosceles right angled
(iii) All sides are of different length	(c) Obtuse angled
(iv) 3 acute angles	(d) Right angled
(v) 1 right angle	(e) Equilateral
(vi) 1 obtuse angle	(f) Acute angled
(vii) 1 right angle with two sides of equal length	(g) Isosceles

3. Name each of the following triangles in two different ways: (you may judge the nature of the angle by observation)



4. Try to construct triangles using match sticks. Some are shown here.
   Can you make a triangle with
   (a) 3 matchsticks?
   (b) 4 matchsticks?
   (c) 5 matchsticks?
   (d) 6 matchsticks?
   (Remember you have to use all the available matchsticks in each case)
   Name the type of triangle in each case.
   If you cannot make a triangle, think of reasons for it.

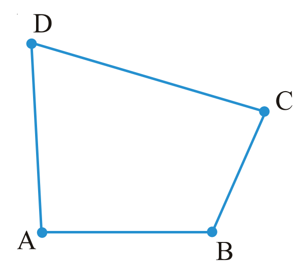


5.8 Quadrilaterals

A quadrilateral, if you remember, is a polygon which has four sides.

Do This

1. Place a pair of unequal sticks such that they have their end points joined at one end. Now place another such pair meeting the free ends of the first pair.
   What is the figure enclosed?
   It is a quadrilateral, like the one you see here.
   The sides of the quadrilateral are $\overline{AB}$, $\overline{BC}$, ___, ___.
   There are 4 angles for this quadrilateral.
   They are given by $\angle BAD$, $\angle ADC$, $\angle DCB$ and _____.
   BD is one diagonal. What is the other?
   Measure the length of the sides and the diagonals.
   Measure all the angles also.



2. Using four unequal sticks, as you did in the above activity, see if you can form a quadrilateral such that
   (a) all the four angles are acute.
   (b) one of the angles is obtuse.
   (c) one of the angles is right angled.
   (d) two of the angles are obtuse.
   (e) two of the angles are right angled.
   (f) the diagonals are perpendicular to one another.

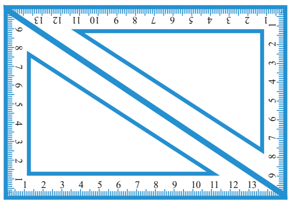
Do This

You have two set-squares in your instrument box. One is 30° – 60° – 90° set-square, the other is 45°– 45°– 90° set square.

You and your friend can jointly do this.

(a) Both of you will have a pair of 30°– 60°– 90° set-squares. Place them as shown in the figure.

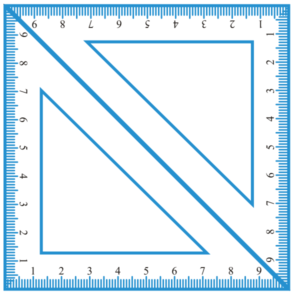
Can you name the quadrilateral described?



What is the measure of each of its angles?

This quadrilateral is a **rectangle.**

One more obvious property of the rectangle you can see is that opposite sides are of equal length. What other properties can you find?



(b) If you use a pair of 45°– 45°–90° set-squares, you get another quadrilateral this time.

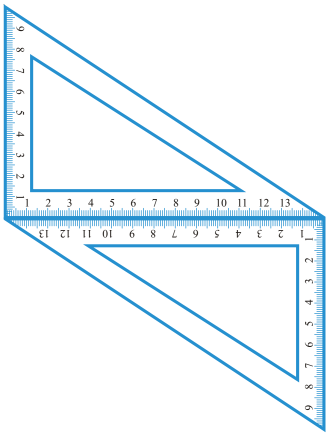
It is a **square**.

Are you able to see that all the sides are of equal length? What can you say about the angles and the diagonals? Try to find a few more properties of the square.

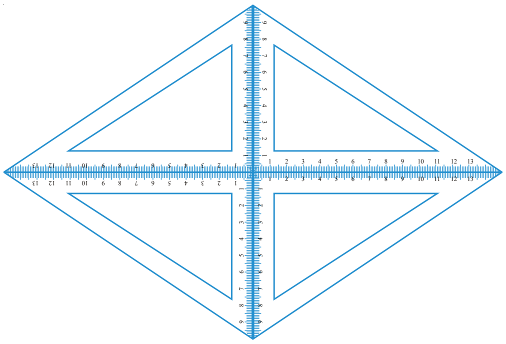
(c) If you place the pair of 30° – 60° – 90° set-squares in a different position, you get a **parallelogram**. Do you notice that the opposite sides are parallel?

Are the opposite sides equal?

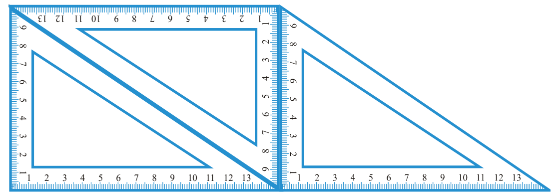
Are the diagonals equal?



(d) If you use four 30° – 60° – 90° set-squares you get a **rhombus**.



(e) If you use several set-squares you can build a shape like the one given here.



Here is a quadrilateral in which a pair of two opposite sides is parallel.

It is a **trapezium.**

Here is an outline-summary of your possible findings. Complete it.

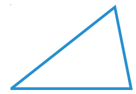
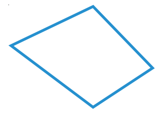
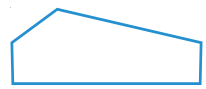
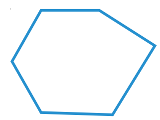
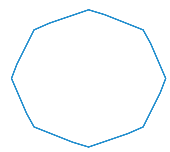
Quadrilateral	Opposite sides		All sides	Opposite Angles	Diagonals	
	Parallel	Equal	Equal	Equal	Equal	Perpen-dicular
Parallelogram	Yes	Yes	No	Yes	No	No
Rectangle			No			
Square						Yes
Rhombus				Yes		
Trapezium		No				

EXERCISE 5.7

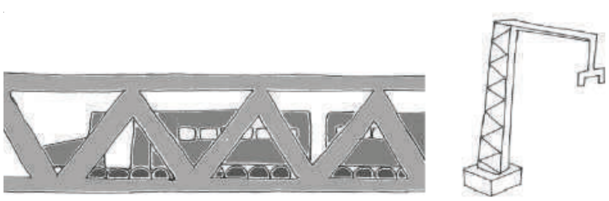
1. Say True or False :
   (a) Each angle of a rectangle is a right angle.
   (b) The opposite sides of a rectangle are equal in length.
   (c) The diagonals of a square are perpendicular to one another.
   (d) All the sides of a rhombus are of equal length.
   (e) All the sides of a parallelogram are of equal length.
   (f) The opposite sides of a trapezium are parallel.
2. Give reasons for the following :
   (a) A square can be thought of as a special rectangle.
   (b) A rectangle can be thought of as a special parallelogram.
   (c) A square can be thought of as a special rhombus.
   (d) Squares, rectangles, parallelograms are all quadrilaterals.
   (e) Square is also a parallelogram.
3. A figure is said to be regular if its sides are equal in length and angles are equal in measure. Can you identify the regular quadrilateral?

5.9 Polygons

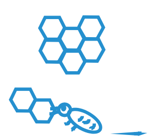
So far you studied polygons of 3 or 4 sides (known as triangles and quardrilaterals respectively). We now try to extend the idea of polygon to figures with more number of sides. We may classify polygons according to the number of their sides.

Number of sides	Name	Illustration
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
8	Octagon	

You can find many of these shapes in everyday life. Windows, doors, walls, almirahs, blackboards, notebooks are all usually rectanglular in shape. Floor tiles are rectangles. The sturdy nature of a triangle makes it the most useful shape in engineering constructions.



The triangle finds application in constructions.

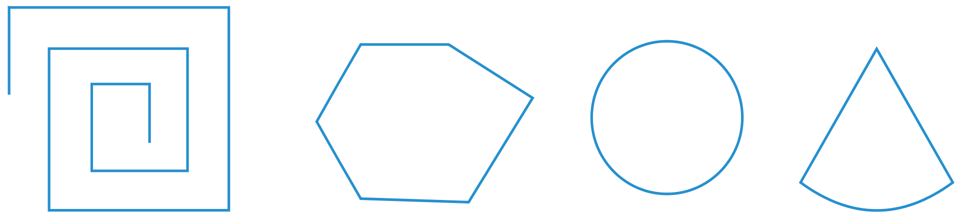


A bee knows the usefulness of a hexagonal shape in building its house .

Look around and see where you can find all these shapes.

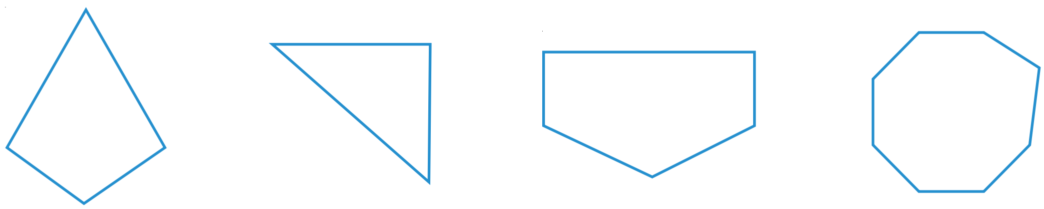
EXERCISE 5.8

1. Examine whether the following are polygons. If any one among them is not, say why?



(a) (b) (c) (d)

2. Name each polygon.



(a) (b) (c) (d)

Make two more examples of each of these.

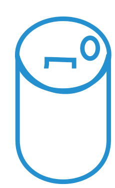
3. Draw a rough sketch of a regular hexagon. Connecting any three of its vertices, draw a triangle. Identify the type of the triangle you have drawn.
4. Draw a rough sketch of a regular octagon. (Use squared paper if you wish). Draw a rectangle by joining exactly four of the vertices of the octagon.
5. A diagonal is a line segment that joins any two vertices of the polygon and is not a side of the polygon. Draw a rough sketch of a pentagon and draw its diagonals.

5.10 Three Dimensional Shapes

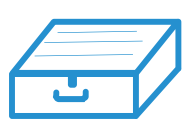
Here are a few shapes you see in your day-to-day life. Each shape is a solid. It is not a 'flat' shape.

The ball is a sphere.

The ice-cream is in the form of a cone.

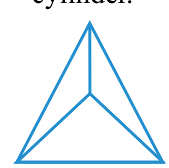


This can is a cylinder.



The box is a cuboid.

The playing die is a cube.



This is the shape of a pyramid.

Name any five things which resemble a sphere.

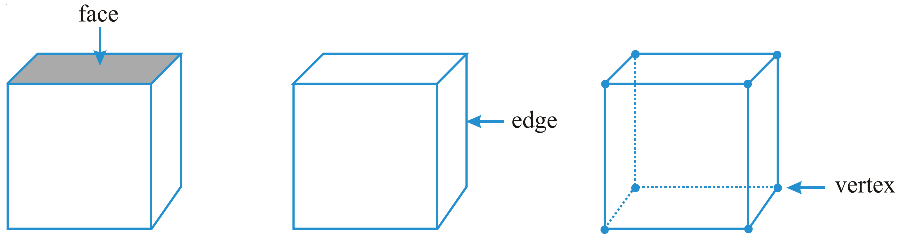
Name any five things which resemble a cone.

Faces, edges and vertices

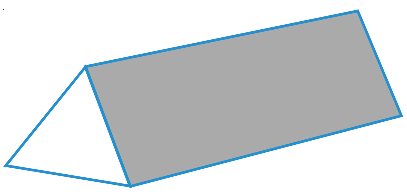
In case of many three dimensional shapes we can distinctly identify their faces, edges and vertices. What do we mean by these terms: Face, Edge and Vertex? (Note 'Vertices' is the plural form of 'vertex').

Consider a cube, for example.

Each side of the cube is a flat surface called a flat **face** (or simply a **face**). Two faces meet at a *line segment* called an **edge.** Three edges meet at a point called a **vertex.**



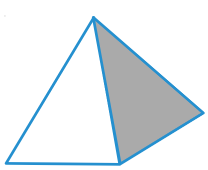
Here is a diagram of a **prism.**



Have you seen it in the laboratory? One of its faces is a triangle. So it is called a triangular prism.

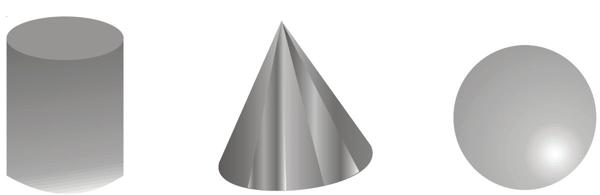
The triangular face is also known as its base. A prism has two identical bases; the other faces are rectangles.

If the prism has a rectangular base, it is a rectangular prism. Can you recall another name for a rectangular prism?



A pyramid is a shape with a single base; the other faces are triangles.

Here is a square pyramid. Its base is a square. Can you imagine a triangular pyramid? Attempt a rough sketch of it.



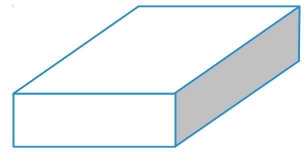
The cylinder, the cone and the sphere have no straight edges. What is the base of a cone? Is it a circle? The cylinder has two bases. What shapes are they? Of course, a sphere has no flat faces! Think about it.

Do This

1. A cuboid looks like a rectangular box.

   It has 6 faces. Each face has 4 edges.

   Each face has 4 corners (called vertices).

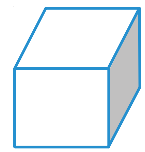


2. A cube is a cuboid whose edges are all of equal length.

   It has ______ faces.

   Each face has ______ edges.

   Each face has ______ vertices.

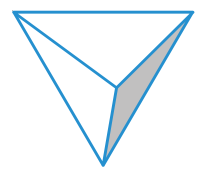


3. A triangular pyramid has a triangle as its base. It is also known as a tetrahedron.

   Faces : _______

   Edges : _______

   Corners : _______

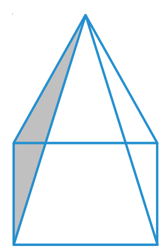


4. A square pyramid has a square as its base.

   Faces : _______

   Edges : _______

   Corners : _______

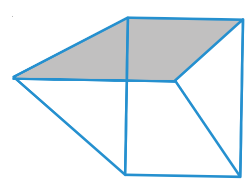


5. A triangular prism looks like the shape of a Kaleidoscope. It has triangles as its bases.

   Faces : _______

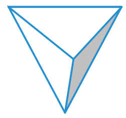
   Edges : _______

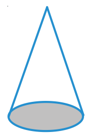
   Corners : _______

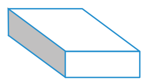


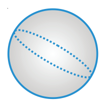
EXERCISE 5.9

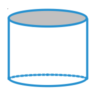
1. Match the following :

(a) Cone (i) 

(b) Sphere (ii) 

(c) Cylinder (iii) 

(d) Cuboid (iv) 

(e) Pyramid (v) 

Give two new examples of each shape.

2. What shape is
   (a) Your instrument box? (b) A brick?
   (c) A match box? (d) A road-roller?
   (e) A sweet laddu?

What have we discussed?

1. The distance between the end points of a line segment is its *length*.
2. A graduated *ruler* and the *divider* are useful to compare lengths of line segments.
3. When a hand of a clock moves from one position to another position we have an example for an *angle*.

   One full turn of the hand is 1 *revolution*.

   A *right angle* is ¼ revolution and a *straight angle* is ½ a revolution .

   We use a *protractor* to measure the size of an angle in degrees.

   The measure of a right angle is 90° and hence that of a straight angle is 180°.

   An angle is *acute* if its measure is smaller than that of a right angle and is *obtuse* if its measure is greater than that of a right angle and less than a straight angle.

   A *reflex angle* is larger than a straight angle.

4. Two intersecting lines are *perpendicular* if the angle between them is 90°.
5. The *perpendicular bisector* of a line segment is a perpendicular to the line segment that divides it into two equal parts.
6. Triangles can be classified as follows based on their angles:

Nature of angles in the triangle	*Name*
Each angle is acute	Acute angled triangle
One angle is a right angle	Right angled triangle
One angle is obtuse	Obtuse angled triangle

7. Triangles can be classified as follows based on the lengths of their sides:

Nature of sides in the triangle	*Name*
All the three sides are of unequal length	Scalene triangle
Any two of the sides are of equal length	Isosceles triangle
All the three sides are of equal length	Equilateral triangle

8. Polygons are named based on their sides.

Number of sides	*Name of the Polygon*
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
8	Octagon

9. Quadrilaterals are further classified with reference to their properties.

Properties	*Name of the Quadrilateral*
One pair of parallel sides	Trapezium
Two pairs of parallel sides	Parallelogram
Parallelogram with 4 right angles	Rectangle
Parallelogram with 4 sides of equal length	Rhombus
A rhombus with 4 right angles	Square

10. We see around us many *three dimensional shapes*. Cubes, cuboids, spheres, cylinders, cones, prisms and pyramids are some of them.